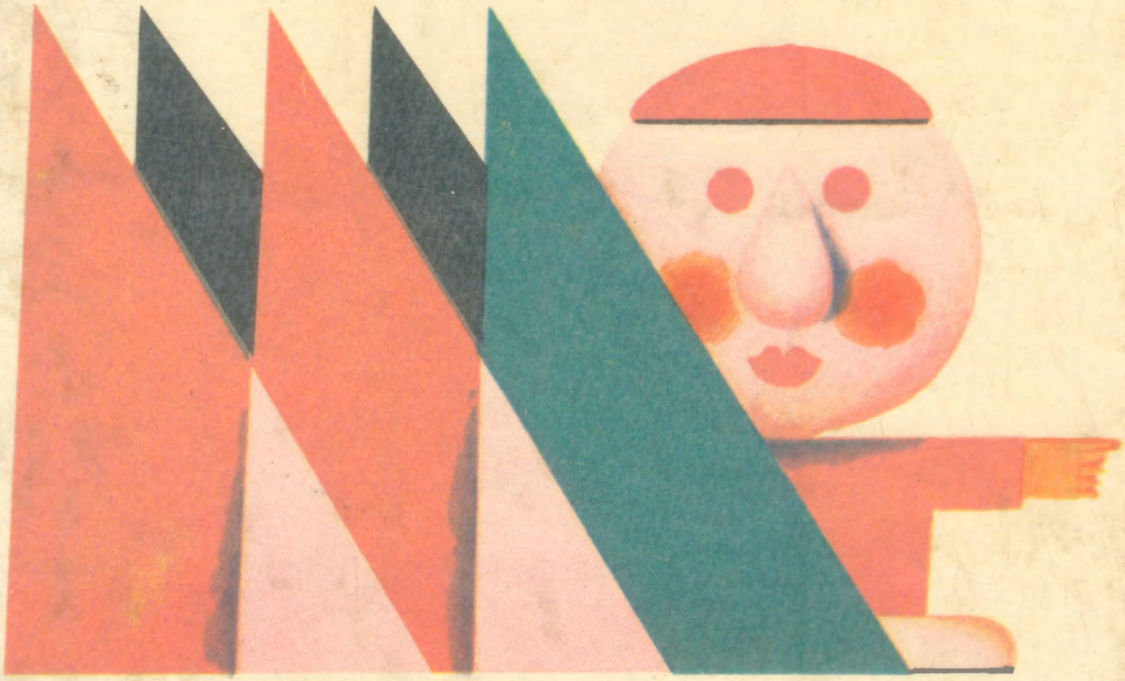
VISIT…

Let's play GEOMETRY

L.N. Shevrin
V.G. Zhitomirsky



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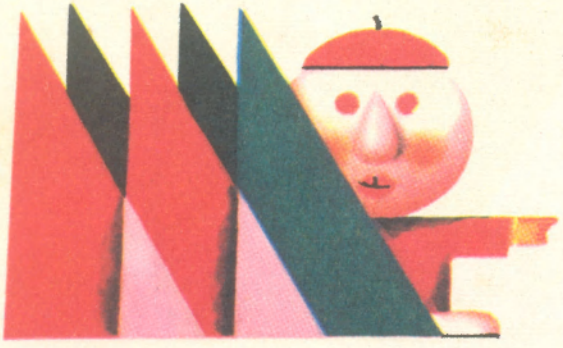
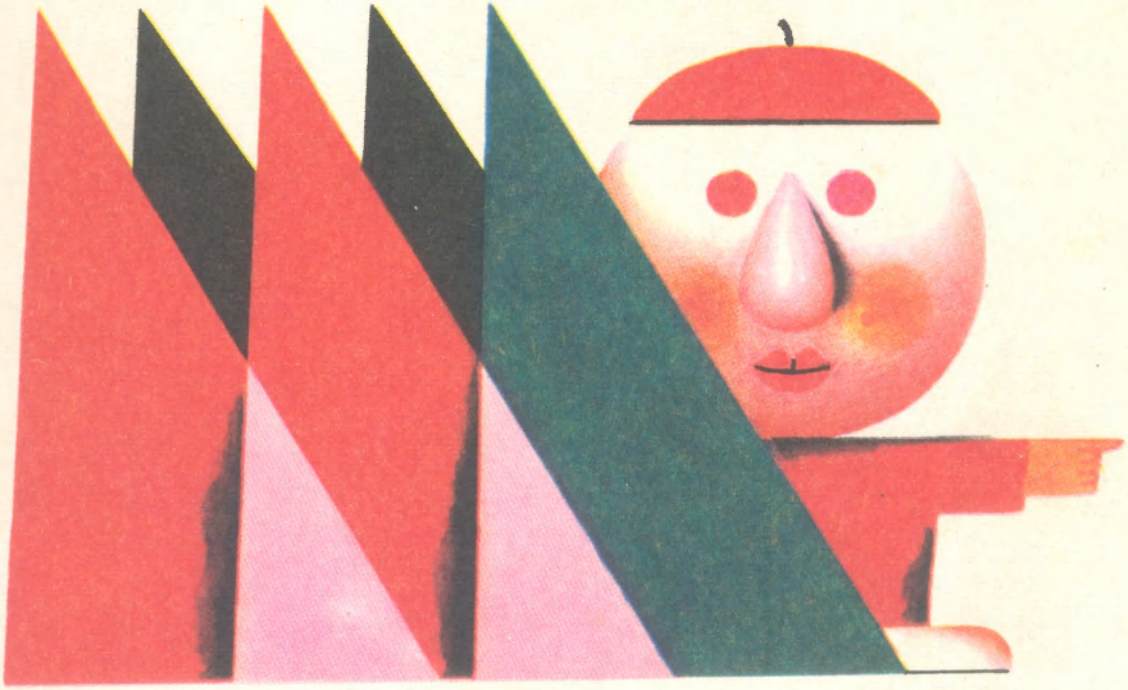


Mir Publishers

GEOMETRY

Translated from the Russian

by Alexander Repyev



Moscow

В. Г. Житомирский, Л. Н. Шеврин

ГЕОМЕТРИЯ ДЛЯ МАЛЫШЕЙ

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На английском языке

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To our children
Igor and Anya Shevrin
Volodya and Sasha Zhitomirsky

TO PARENTS
and Grandparents and the Adults Who Will Read This Book to Children

It is well known that authors always write prefaces after the body of the book has been written. We are no exception to the rule and it was only after the last page was ready that we set to the task of producing the preface. But even before the first page had been turned out we knew that we would have to formulate quite clearly the major points to be included in the preface. After all, the preface is the right place for suggesting guidelines for those who would use the book to teach children. We made a point of putting down in a separate notebook the ideas worth reflecting in the preface. But when, the work over, we reread the notebook all the way through, we realized that just listing these "ideas" without expanding them would produce a "consistent" piece of preface. So we reproduce them here as initially compiled:

1. The book is designed for children from 6 to 8 years old, but it might be of interest both to five-year olds and to older primary-school pupils.
2. The book can be used in any of the following ways: family reading, group reading in Infants schools, and recreational reading in the lower Junior schools.
3. The book is no study-book. It does not provide a systematic and complete introduction to geometry. Its aim is to acquaint children in an exciting and stimulating way with some of the principal concepts of geometry, to teach them how to find bearings in simple geometrical situations and to discern geometrical patterns in the world around them.
4. Despite its "easiness", the book contains some fundamental scientific facts. Therefore, using the book calls for active involvement on the part of adults. At times it may be necessary to expand on those points that appear to be difficult, and to comment on the drawings and pictures emphasizing some details.
5. Since the book contains many new ideas and facts, it should be read step by step and the "dosage" should be reasonably graded. Granted the reading "doses" are substantially dependent on the individual ability of a child, but according to our experience a daily session should not be longer than 30-40 minutes (especially with group reading).

6 Each term for a new concept, when it first appears in the text, is printed in red. It would pay to stop in those places, concentrating on the new term, repeating it several times, checking and rechecking the response. It does not matter if a child has not remembered all the new words and definitions at once. What is more important is that he or she listen attentively and understand.

7 It is a good practice to begin each session with recapitulation: recall the previous events and the definitions of the earlier concepts.

8 The tasks and the worksheets are important. They will make for a deeper understanding of the mathematical contents of the book and development of some practical skills with geometrical figures. Children should not overlook these questions and exercises.

9 Some of the exercises are far more difficult than those inserted into the body of the text. Some even introduce new concepts. It is up to the instructor to decide whether or not the student can handle them and they should not insist on carrying out the exercises in strict succession, especially if this will generate "adverse emotions" in the students.

10 Your sessions will require coloured pencils, paper, a ruler, a pair of scissors, a pair of compasses, a set-square, sticks, and some Plasticine. All these should be prepared beforehand and used as required.

11 With group reading, some sort of competition could be stimulated (who will be the first to answer a question, or handle a task), or discussions conducted, and so on. Such possibilities are not to be overlooked!

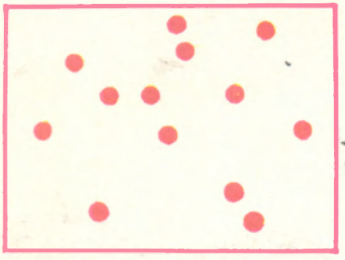
12 Enjoy the book!

The Authors

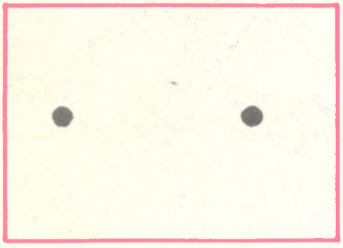
Moscow, 1984

Pencil once invited the Happy gang round to tea. The Happy gang were his friends Pinocchio, Dunno, and Gadgit. After tea he said:
"Let's do some geometry. It's very interesting."
"Let's!" said Gadgit and Dunno, but Pinocchio asked:
"What's geo-mer-ty?"
"Not geomerty, but geometry," said Pencil. "Geometry – it's... it's... I can't explain it at once. Let's get started, and bit by bit you'll learn everything."

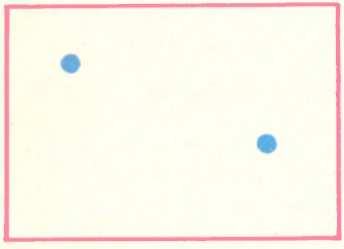
The friends sat round the table.
"Well, look," said Pencil and pecked a sheet of paper with his nose.
"What's it?"
"A point," said Gadgit.
"A point," echoed Dunno.



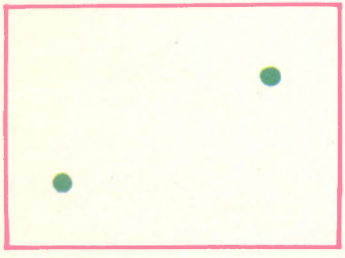
But Pinocchio said nothing, he just poked his nose into some paint and drummed with it quickly at a sheet of paper... rat-a-tat-tat.
"And I've got many points!" cried out Pinocchio.



"Don't hurry," Pencil stopped him and put down another point on his sheet.
"Now I've made two points."



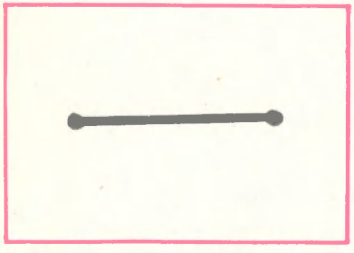
"Two points," muttered Dunno and also drew two points on his paper.



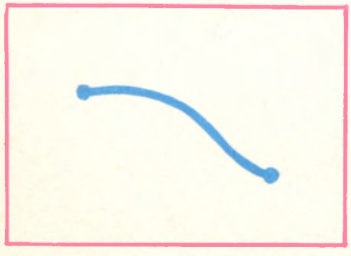
Gadgit also drew two points.

Now you draw two points on your sheet of paper.

Many points came to live
on Pinocchio's paper leaf

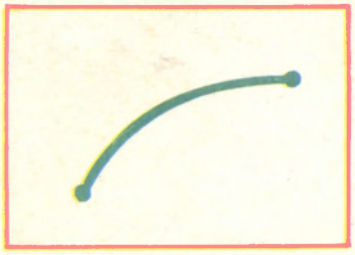


Pinocchio started to sing but was hushed up by Pencil.



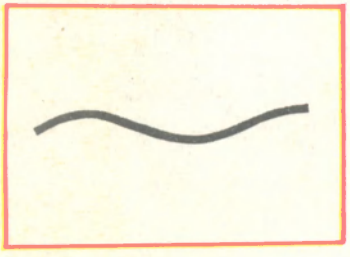
"Now I connect the points," said Pencil. "And I've got a line. You do the same."

Dunno did this:

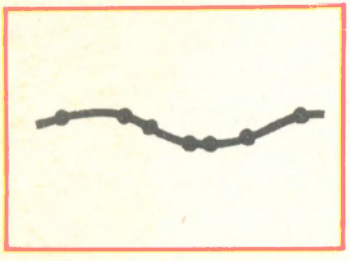


Gadgit this.

Now you connect your points by a line.

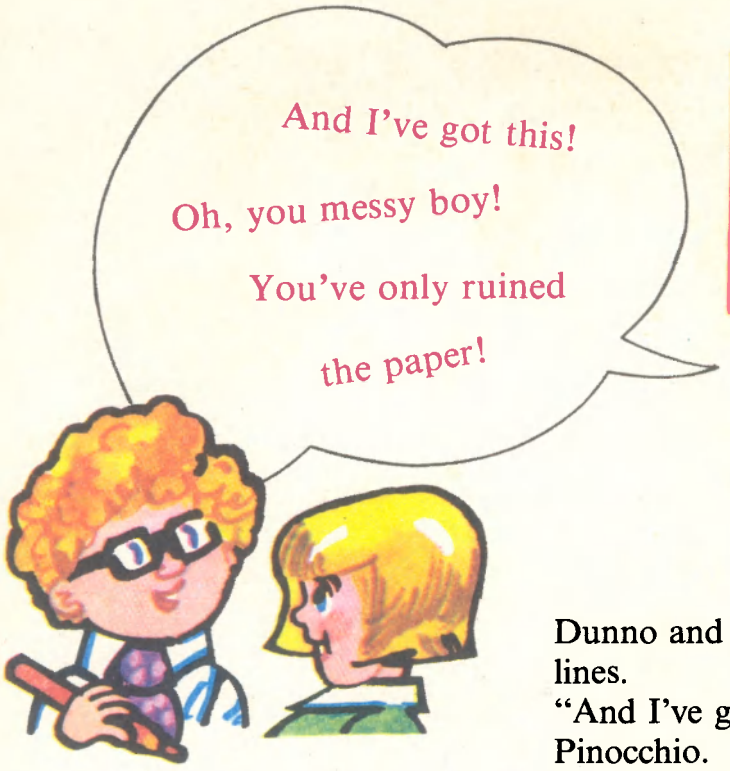


"But is it possible to draw a line without drawing two points at first?" asked Dunno.
"Of course," said Pencil and drew a new line.



"So it is a line without points?" again asked Dunno.
"No, why! A line is all made up of points. At any place along a line you can mark a point. Just look, I've marked several points on my line."

Now you draw a line and mark on it several points.



Dunno and Gadgit also drew their lines.

"And I've got this!" cried out Pinocchio.

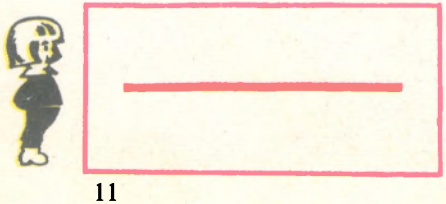
"Oh, you messy boy!" Gadgit shook his head. "I can't make out anything. You've only ruined the paper!"

"Yes," said Pencil. "We'll have to take the paint away from you. Here is a red pencil and a blank sheet of paper. Now draw a line. Look, Gadgit made a straight line."

Pinocchio did his best.

"But mine isn't as smooth as Gadgit's," he was distressed.

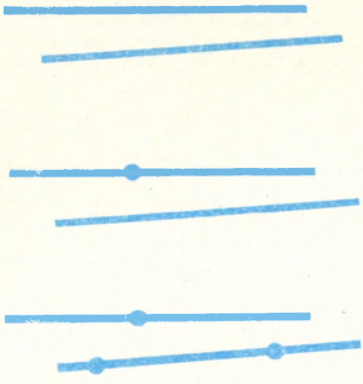
"You take a ruler," said Gadgit, "press it down onto the paper and draw the pencil along it."



"I did it!" Pinocchio was happy. "It's so smooth!"

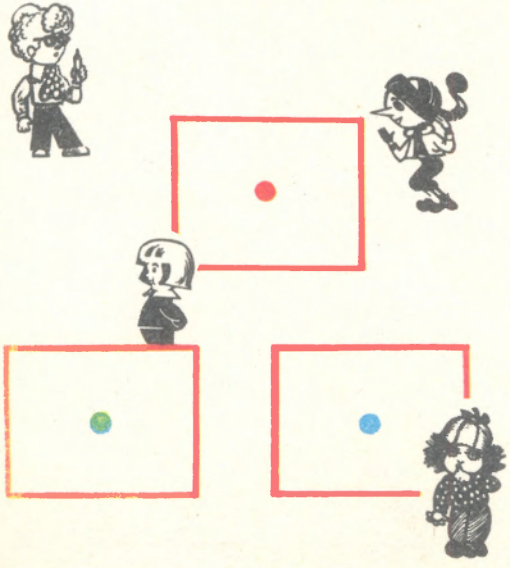
"This is called a straight line," explained Pencil.

A long-long straight line,
Drew the pencil of mine,
My ginger line is fine,
And my nose is of pine.



"Please, give me a ruler," said Dunno. "I also want to draw straight lines. I've drawn, I've even got two straight lines."
"Well done," said Pencil. "Now put a point on the upper line."
"Done it."
"And on the lower one, mark two points."
"Done it again," Dunno was pleased.

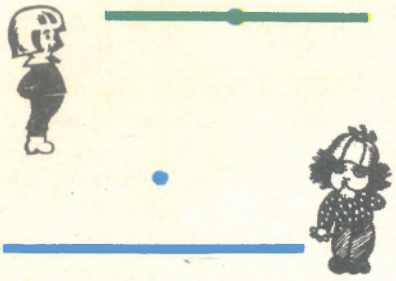
Now you take a ruler, draw straight lines and mark points on them.



"Here's a more difficult problem," said Pencil.
"First put down a point and then draw a straight line through it."

To put down a point is easy, but to draw a straight line through it is more difficult.

Here is what Gadgit got:



And Dunno:

Say which one managed to do the task?

But Pinocchio was only turning his head around and making fun of Dunno, although he himself hadn't done anything yet.

"Ha-ha, he couldn't make it, Dunno couldn't make it."

"Yes," said Pencil. "Dunno, your point is above the line. And, Pinocchio, don't you laugh. You haven't done anything. You just try and draw a straight line through the point correctly."

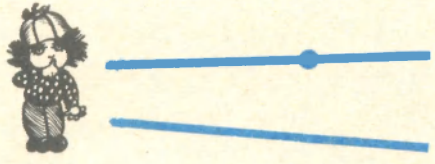
"Okay!" cried out Pinocchio. "It's a piece of cake."

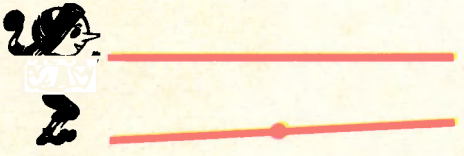
And he drew his line thus:

"Aha," smiled Dunno. "You laughed at me and you failed too. Your point is not on the line."

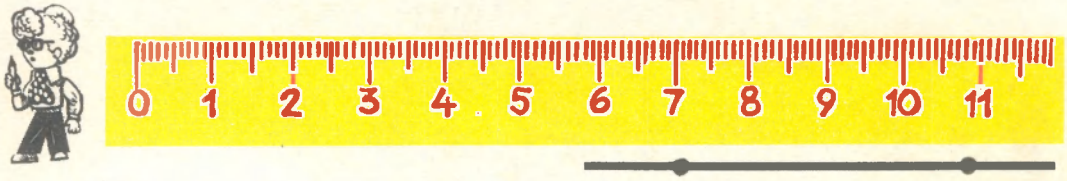
"Your point, Pinocchio, is under the straight line," Gadgit added.

So Dunno and Pinocchio had to draw their lines again and this is what they got:



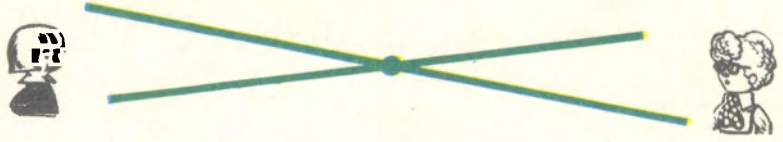


Then Pencil showed them how to draw a straight line through two points:



Now you mark a point, take a ruler and draw a straight line through the point. Then mark two points and draw a straight line through them.

Gadgit quickly drew two straight lines through one point and showed his job to his friends.
"Look," said Pencil, "the straight lines Gadgit drew intersect."

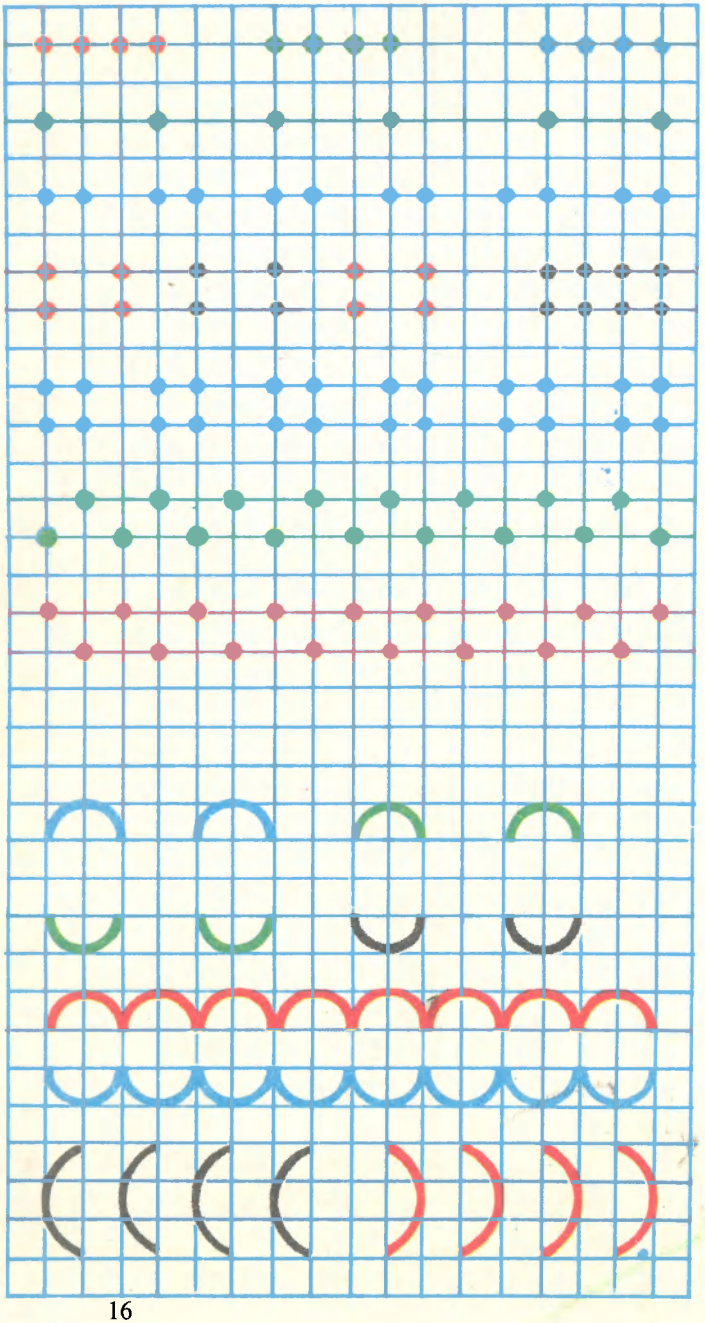


Show the point at which these straight lines intersect.

"My lines intersect too," Pinocchio said quickly.
And here are some other intersecting lines:

Show the points at which they intersect. How many intersection points have these lines? Now you draw intersecting lines and note their intersection points.

WORKSHEET

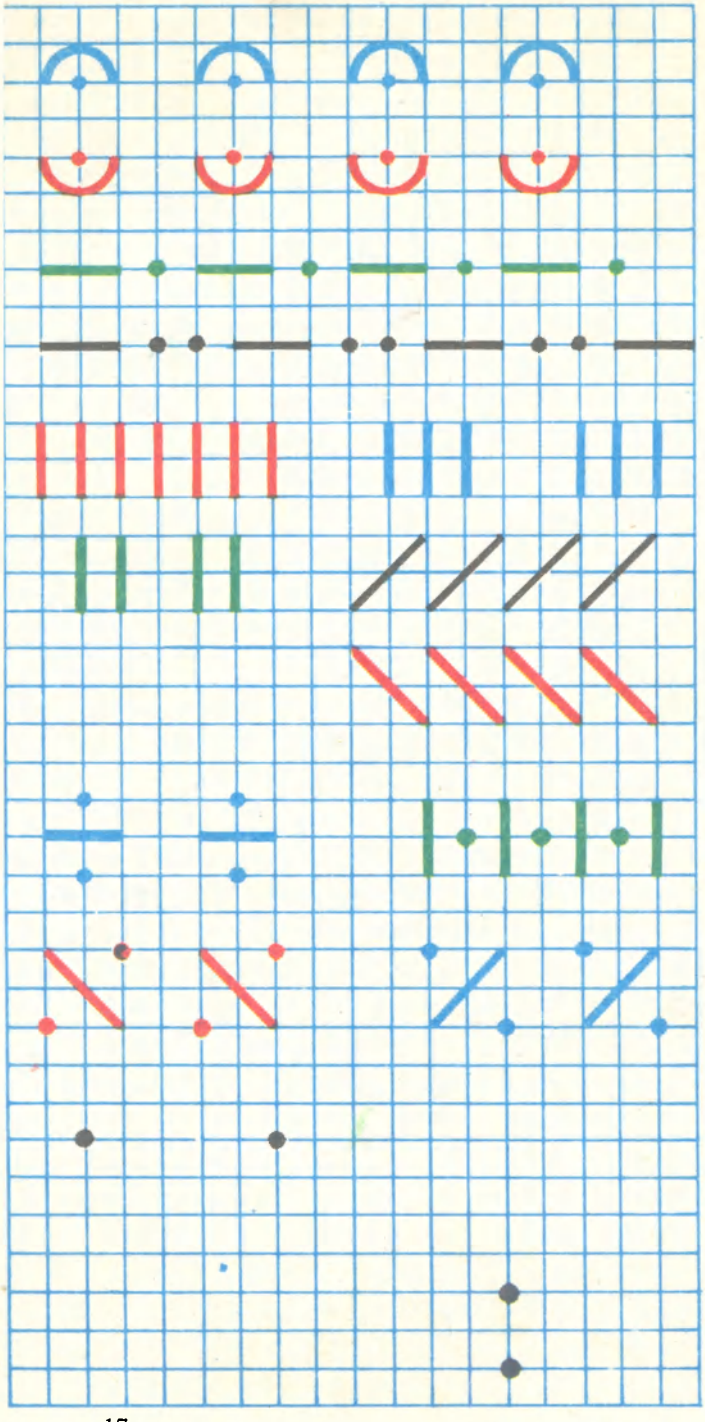


1

Draw:

2

Draw such lines:

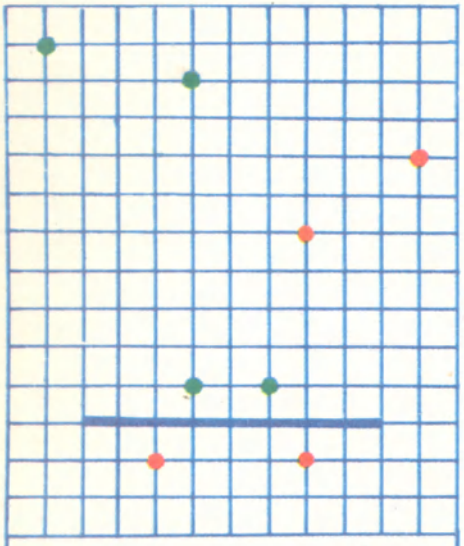


3
Draw:

4
Draw such lines:

5
Draw:

6
Mark two points thus:
Now take a ruler and draw a straight line through them.
And now through these two points:



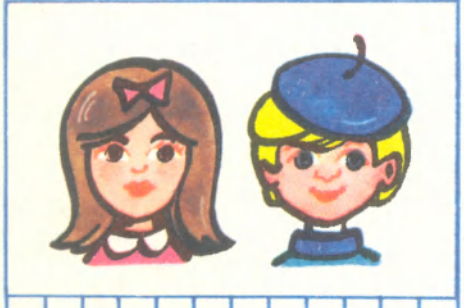
Through these:
And through these:

7

Which points lie above the straight line, and which lie under it?

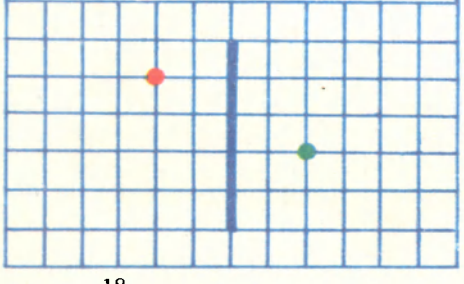
8

Look at the stool and chair. The stool is to the left of the chair, and the chair on the right of the stool. And here stand a boy and a girl. Who is on the left and who is on the right?



9

Lift your left hand, then your right hand. Stamp your right foot, then your left foot.



10

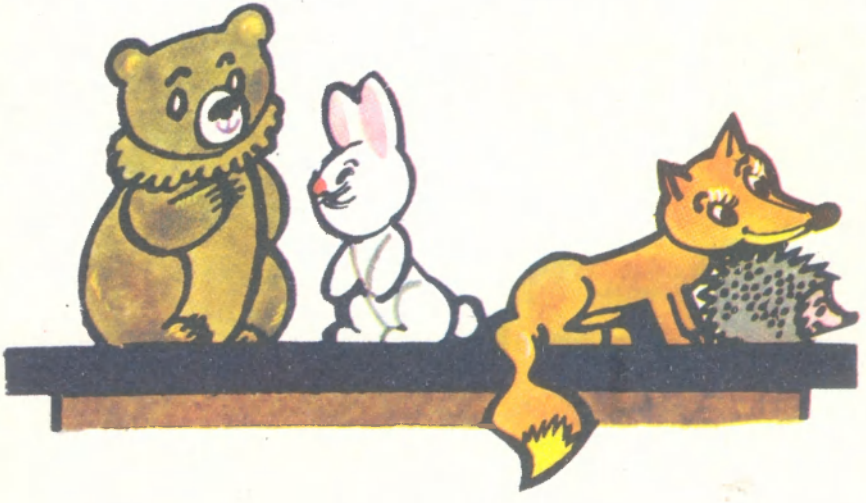
The two points lie on either side of the straight line. Point out which one is on the right and which is on the left.

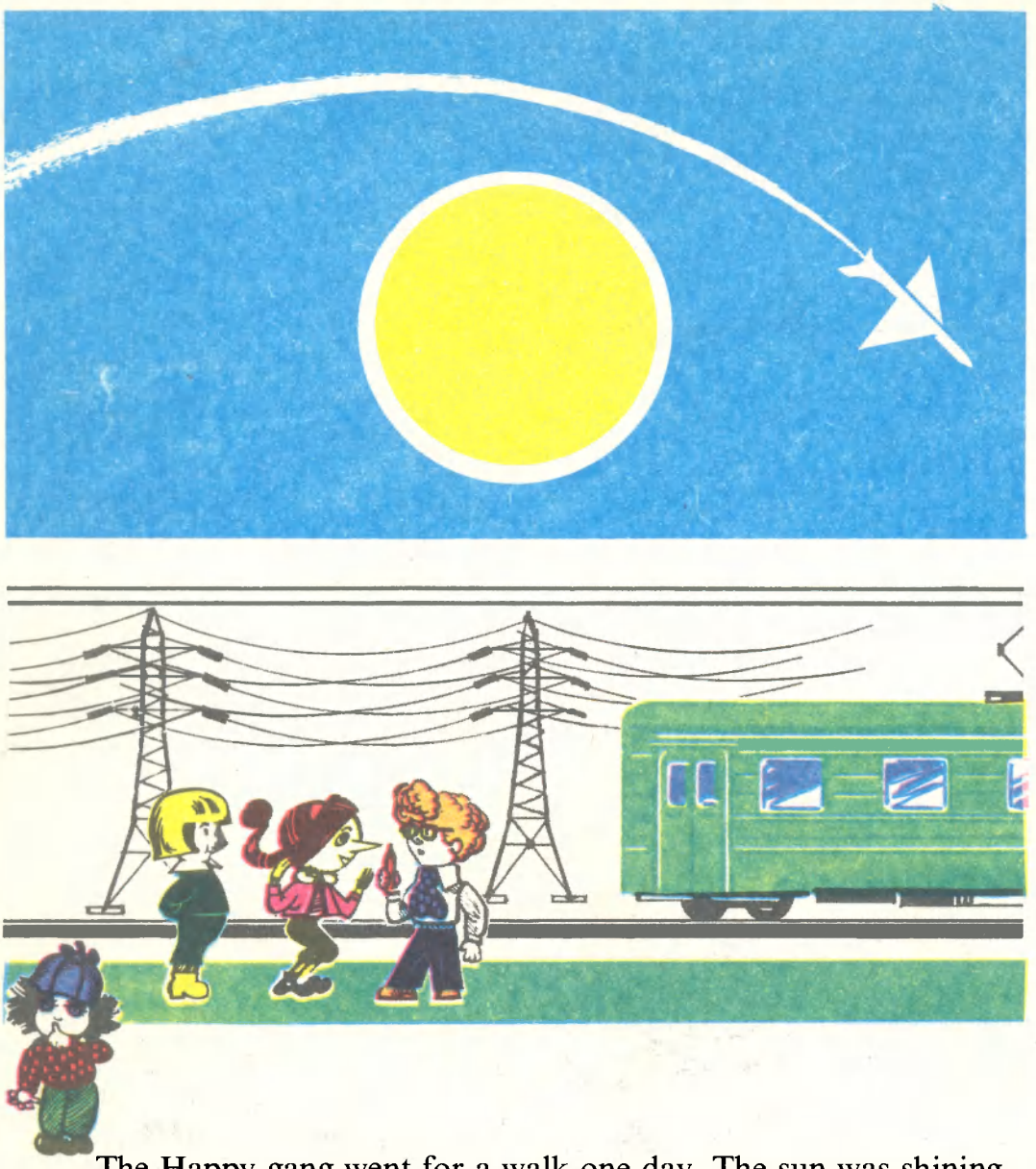
11

A spruce, a pine and a birch-tree grow side by side. The pine is between the birch-tree and the spruce. Which tree is on the right of the pine? Which trees are on the left of the birch-tree?

12

There are some toys on the table: a teddy-bear, a hare, a fox, and a hedgehog. Which toy stands between the hare and hedgehog, between the teddy-bear and hedgehog? Which toys stand on the left of the fox, and which on the right of the teddy-bear?





The Happy gang went for a walk one day. The sun was shining brightly in the blue sky over them. Very high in the sky, almost near the sun, a jet plane was flying, leaving a white trail behind it. When Gadgit saw the trail, it seemed to him that it was a white pencil drawing on a huge blue sheet of paper.

"Look," he cried, "what an interesting line the plane has drawn in the sky!"

Dunno suddenly wanted very much to show his friends a line too. He looked up and down, to the right and to the left, but he couldn't find a line.

"Uh, it looks like there aren't any more lines around," he sighed.

"Just look over there," said Pencil.

"Oh, the wires," Dunno was delighted. "They are lines, too."

"Right," agreed Pencil. "These wires are straight lines. You see, they are stretched taut, but those wires over there are sagging, so they are curved lines, not straight lines."

Here Pinocchio grinned and said mysteriously:

"Look at me. I'll show you something. I've got a piece of string in my pocket. If I throw it... Hup... there is a curved line.

Dunno, take one end of the string and hold it. I'll take the other end and we stretch the string."

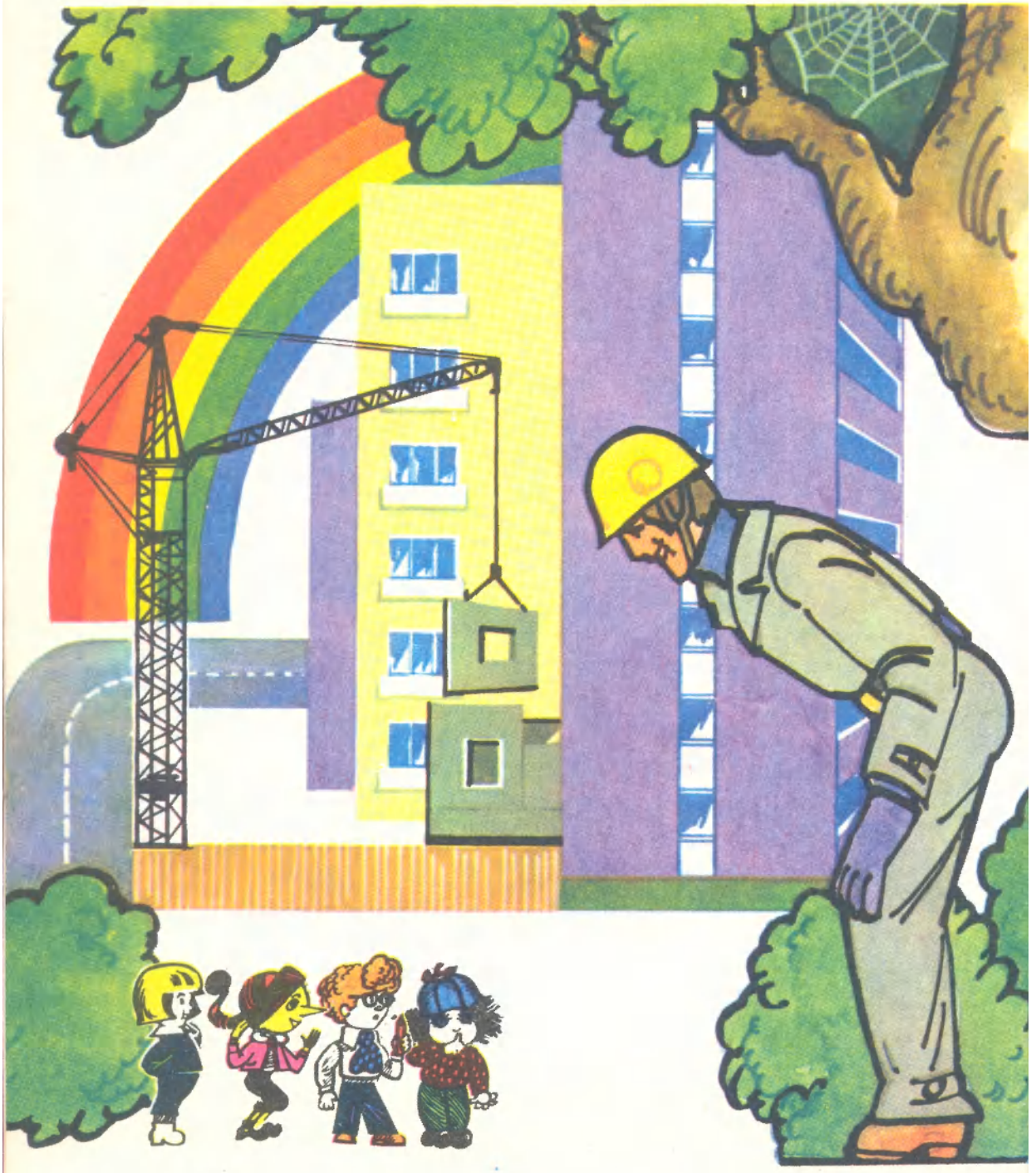
Pencil congratulated Pinocchio:

"Well done! A good idea. Friends, let's look and see what other lines there are around us."

This is a straight line.

You can make any kind of line with this piece of string.

Now you take a piece of string and make into various lines.



The Happy gang began to look around and saw some interesting things.
Railway tracks ran along as straight lines and at bends they were curved lines.
Rain stretched down transparent merry lines to the ground.
A wide arc of a rainbow crossed the sky.

Can you name the colours of the rainbow?

And close at hand, between the branches of a tree a web was hanging. Its thin web-lines, intersecting one another, made an intricate beautiful pattern.

Well, what lines do you see about you? Which of them are straight?

The Happy gang went on along the road. Near the road, stood a house, or rather half a house. Two storeys already stood above the ground and the builders were making the third one. A crane was helping the builders. It was lifting large blocks from the ground and giving them to the builders. The steel rope was taut, stretched by the weight.
"Another straight line," Gadgit pointed at the rope. "It goes straight down."
"Such a line is said to be vertical," explained Pencil.
"Vertical," repeated Dunno.
"Yes," said Pencil, "a vertical line goes exactly down from above or exactly up from below. So if you take one end of a piece of string, and suspend a weight on the other, then the string will hang vertically."
And Pencil looked at Pinocchio, "Where is your string?"
"Just a minute... I'll put a stone on it... ready," said Pinocchio.

He held up the string with a stone tied to it and sang:

Here's stone and here's string,
Tie them up and they'll swing,
Wait until they stop awhile.
Then you'll see a vertical line.

"A good song, Pinocchio," came an unknown voice. A builder stood nearby smiling kindly.

"Well, you know that we, builders, often use such strings with a weight in our work."

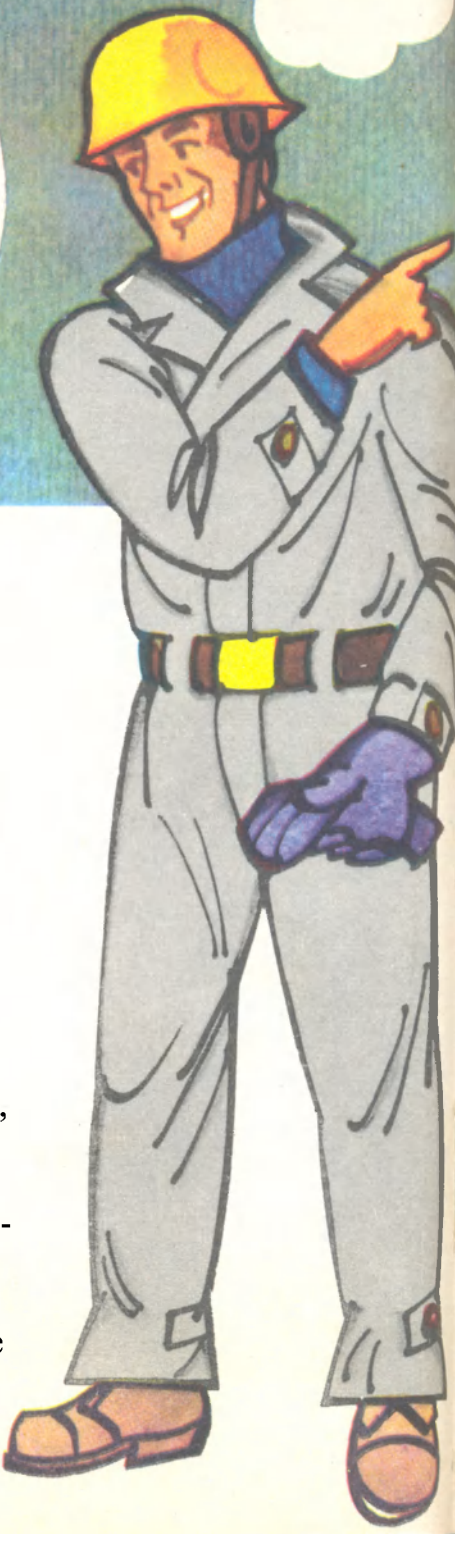
"What for?" asked Pinocchio.

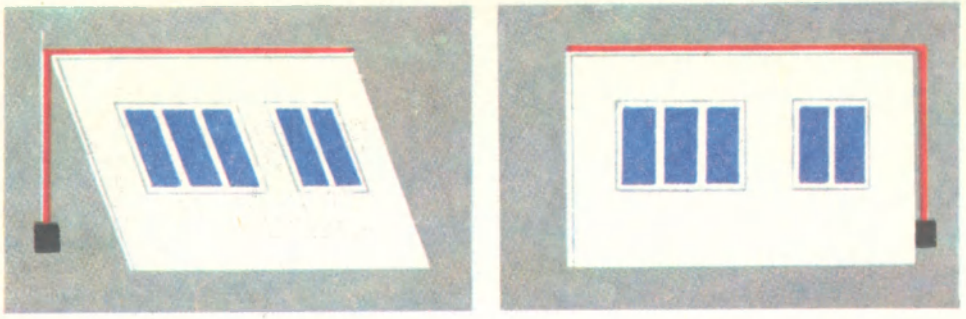
"In order to check if the wall of a house is standing vertically, and isn't leaning over."

"But how can you do that?" asked Gadgit.

"Well, if the wall did lean over, the string would go not along the wall, but like this or that: Builders have to erect walls vertically, like this:

"But not only do walls of houses have to be vertical," the





worker went on to say. "And chimney stacks, and lamp posts.' "And trees also grow vertically," Dunno pointed at a high pine.

"Not every tree grows vertically," said the builder. "Look at those trees. You see, they are leaning, and you can easily check that with your string."

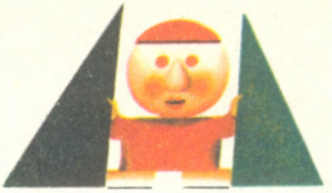
?

Now you take a piece of string, tie a weight to it and check to see if the legs of a table, or a chair, the door of a closet, a door of a room are vertical or not. What vertical and inclined lines do you see around?

When the Happy gang parted with the builder, Dunno timidly asked Pencil:
"Are there any fairy tales about geometry? I like fairy tales so much."
Gadgit laughed:
"Fancy Dunno, asking for a fairy tale. Just like a kid. What fairy tales do you think can be in such a serious business! It's geometry."
"Ha-ha," agreed Pinocchio, "the little Dunno wants a fairy tale. Ha-ha."
"You are wrong, Gadgit," said Pencil, "I just happen to know a geometrical fairy tale. Shall I tell you it?"

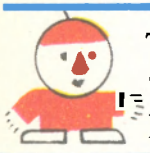
"Yes," Pinocchio was the first to cry out.
"Of course, we want to listen to your tale," said Dunno, "I would like it so much to listen to geometrical stories." Pleased, he turned to Gadgit: "You see, and you teased me."
Gadgit shrugged silently, but it was clear that he too wouldn't mind listening to the tale.
"Well, listen," said Pencil, "my fairy tale is called ..."

POINT'S
Travels in Geometry Land



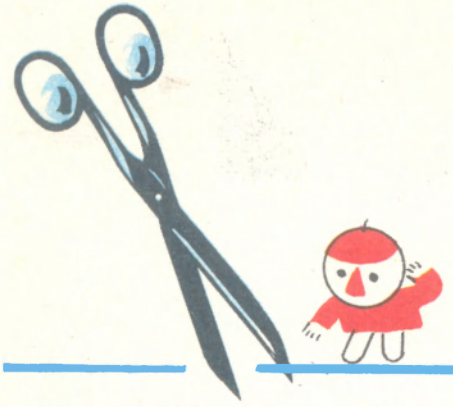
Once upon a time there lived Point. She was very curious and wanted to know everything. If she saw a line you could trust her to ask:
"What do they call it? Is it long or short?"
So once she thought: "How can I possibly learn everything if I am a stay-at-home? I'd better set out on a journey."
No sooner said than done. And Point started walking along a straight line. She kept walking for a long time and got tired.

She stopped and said:
"How long shall I go on? When will this straight line end?"



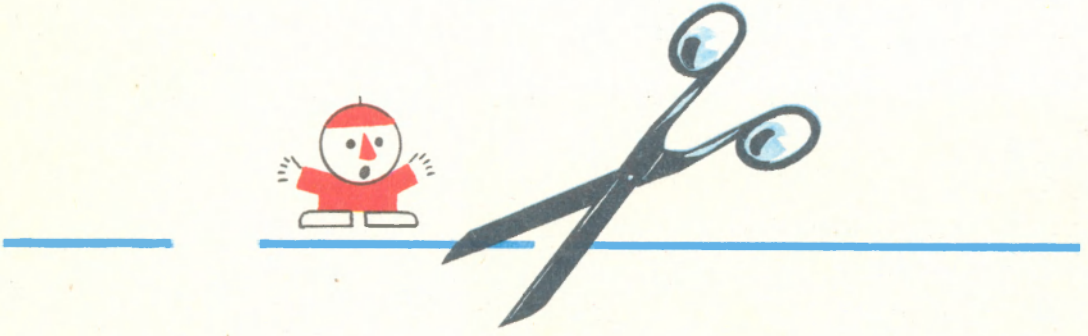
The straight line laughed:
"You silly girl! You will never come to an end, didn't you know that a straight line has no end?"

"Then, I'll turn back," said Point. "It seems I went the wrong way."
"In the other direction, too, you will find no end. A straight line has no ends at all."
Point was sad:
"What shall I do? Will I have to go on and on for days on end?"
"Okay," said the line, "if you don't want to walk endlessly, let's call a pair of scissors to help you out."



"Let's," Point was glad, "but why do we need the scissors?"
"You'll see," said the straight line.
The scissors appeared suddenly, clapped in front of Point's nose and cut the line.

"Hurray!" yelled Point. "Now we've made an end. There are good scissors. Now, will you please make an end on the other side."
"Okay," said the scissors and snapped obediently.
"How very interesting!" cried Point. "What have you made on my straight line? An end on either side. What is this called?"
"This is a segment," said the scissors. "Now you, Point, stand on a segment."

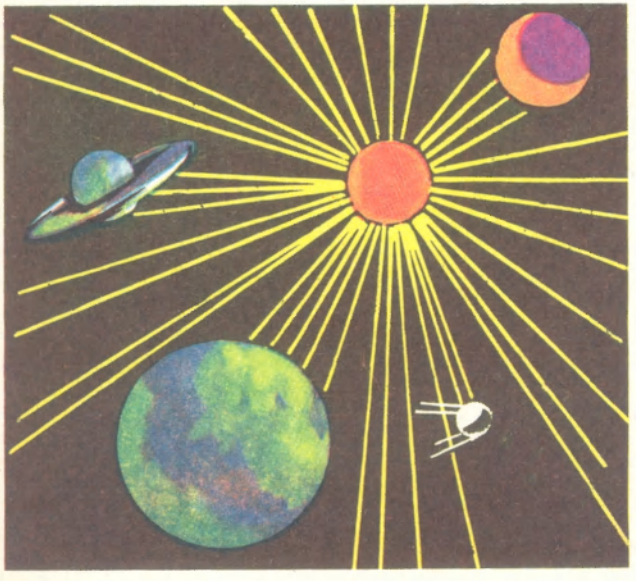


"A segment, a segment," repeated Point merrily, walking from one end to the other. "I'll remember this name. I like it here on the segment. But I liked the straight

line too. It's a pity that it is no more. Now instead of the straight line I have my segment and two of these... don't know what they are called. Also segments?"
"No," said the scissors, "look, they only have an end on one side, and no end on the other side. And they are called differently."
"What are they called?"
"Rays."

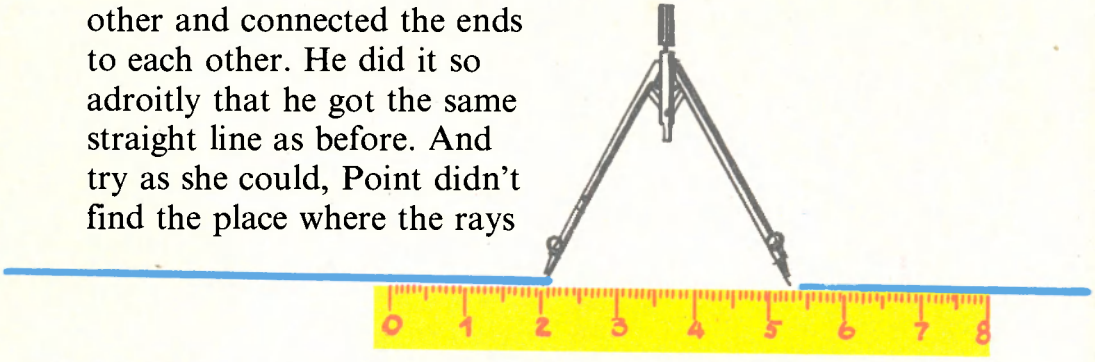
This is a ray and this is a ray.

"Aha," Point cried out, "I know why they are called that. They are like the sun's rays."
"Yes," agreed the scissors. "The sun's rays begin at the Sun and travel endlessly, if only they don't meet something on the way. For example, the Earth, the Moon, or a satellite."

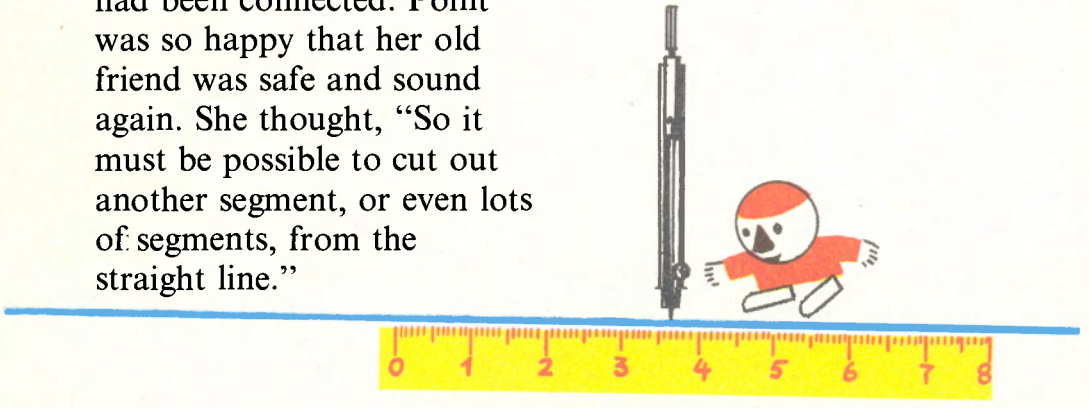


"So the straight line gave me a segment and two rays. But the straight line has gone. Dear scissors, will you please make the straight line again. Only so that my segment remains."
"Oh, I cannot do this. If only I call a pair of Compasses and a ruler for help."

The scissors called in his assistants. The ruler and Compasses came and set about their job. At first Compasses attached to the ruler one ray, then the other and connected the ends to each other. He did it so adroitly that he got the same straight line as before. And try as she could, Point didn't find the place where the rays



had been connected. Point was so happy that her old friend was safe and sound again. She thought, "So it must be possible to cut out another segment, or even lots of segments, from the straight line."



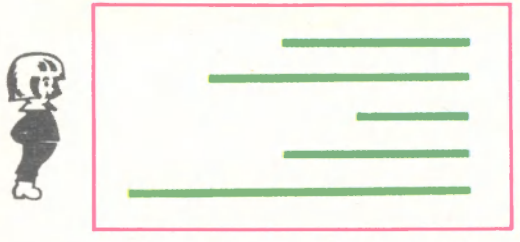
She got the scissors to cut out many different segments, short and long, from the straight line. And Compasses and the ruler connected the remaining rays. Again, the straight line appeared to be safe and sound.

"How is it," Pencil stopped his story, "do you like the tale?"
"We liked it," cried Pinocchio. "I even made up a song about the straight line":

You cannot bound or confine
You cannot know or twist this line.
Along it walk for days on end
You won't find a single bend.

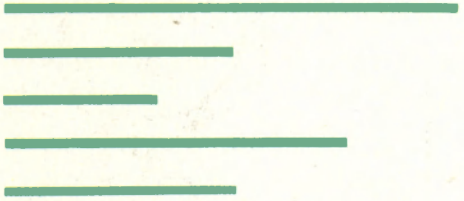
"I also wanted to make up something about segments, only I hadn't time."
"Segments? Gadgit is drawing them," said Dunno pointing.

Indeed, the diligent Gadgit had found somewhere a sheet of paper and a ruler, and was drawing. Here are the segments that Gadgit had drawn:

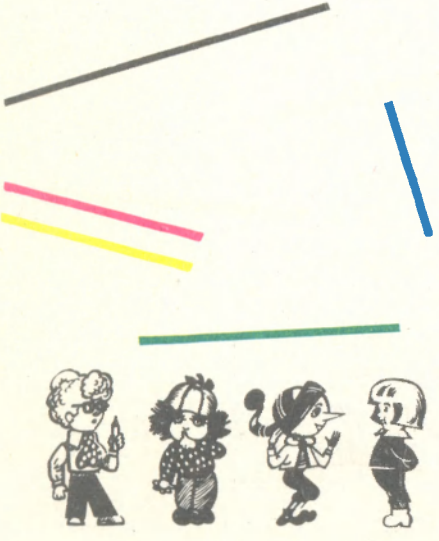


Now you take a sheet of paper, a pencil and a ruler, and draw as many segments as Gadgit did. Count the segments you've drawn.

"Look here, Gadgit, your segments have different lengths," said Pinocchio.
"I did it specially," said Gadgit, "show me the shortest segment."
"This one," Pinocchio had found it quickly, "and this is the longest."
"But these two segments are the same length, aren't they?" added Dunno.



Now you point to the shortest and the longest segment Gadgit has drawn. Find segments that are equal in length. Now draw some segments of your own.

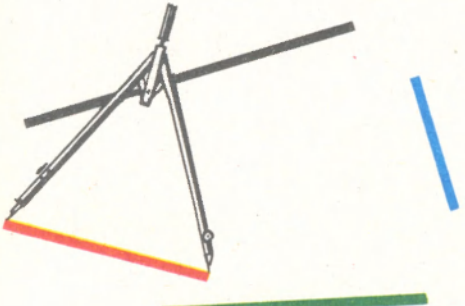


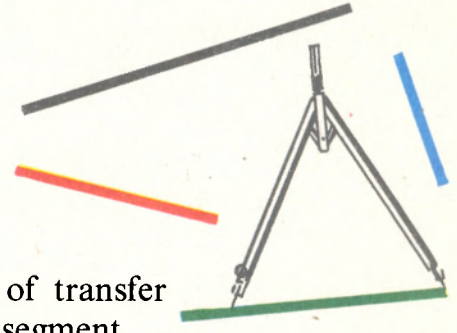
"Good boys!" said Pencil. "Now we will take a more difficult problem. Gadgit, draw some segments but at random, not one under another."
"Why Gadgit again? I also want to draw," cried Pinocchio.
"So do I," said Dunno.
"Well," said Pencil, "let each of you draw a segment."
"Let's see," Pencil went on to say, "these segments are more difficult to compare. How can we find the shortest and the longest?"
"I've found the longest," said Pinocchio, "it's the red one."

"No, the blue one is the longest," said Dunno.
"This way we can argue till morning," said Gadgit, "the segments are about the same length. And you cannot tell by eye alone which is the longest and which is the shortest. We need a more accurate way. Only I don't know such a way... What's to be done?"

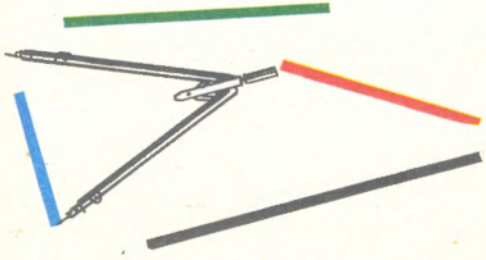
? Can you determine exactly which of these segments is the longest and which is the shortest?

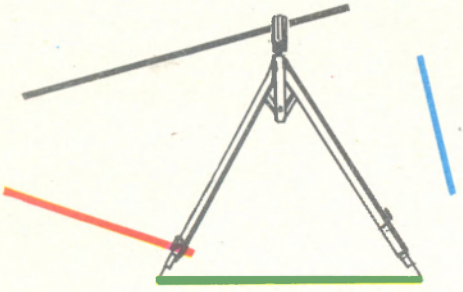
Gadgit, Pinocchio and Dunno looked hopefully at Pencil: he was sure to know what was to be done.
Indeed, the clever Pencil did know that a pair of dividers was needed here. So he explained to his friends how to compare the segments using dividers and thus be able to tell which was longer and which was shorter.
"For example, let's compare the red segment with the blue one," he said. "Put the dividers across the red segment. Now

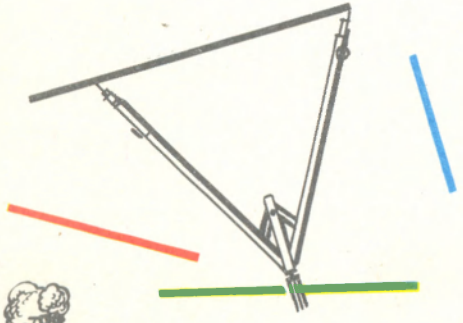




we sort of transfer the red segment to the blue one. The arms of the dividers must be left as they were. It should be clear to everybody now that the red segment is longer than the blue one."







"I told you the red one is the longest," said Pinocchio and looked at Dunno triumphantly.

"Perhaps you are rejoicing a bit early?" said Gadgit. "We haven't yet compared the red, green and black segments. Let's see. You see now, Pinocchio, the red segment is shorter than the green one. You were wrong."

"Perhaps I was right," said Dunno timidly. "Is the green segment the longest? Compare it with the blue and black ones."

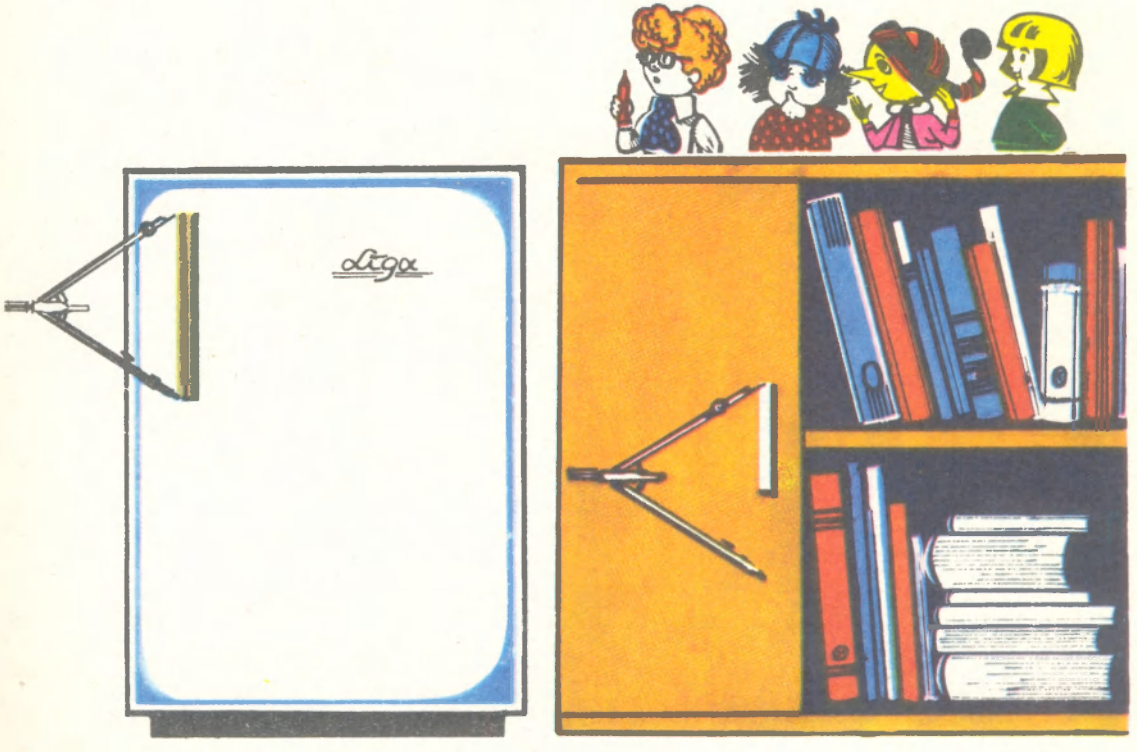
"Well, we needn't compare it with the blue segment," said Pencil. "The green segment is longer than the red one, and the red segment is longer than the blue one. Hence the green is even longer than the blue. So we only have to compare it with the black segment. We'll apply the dividers across the green segment.

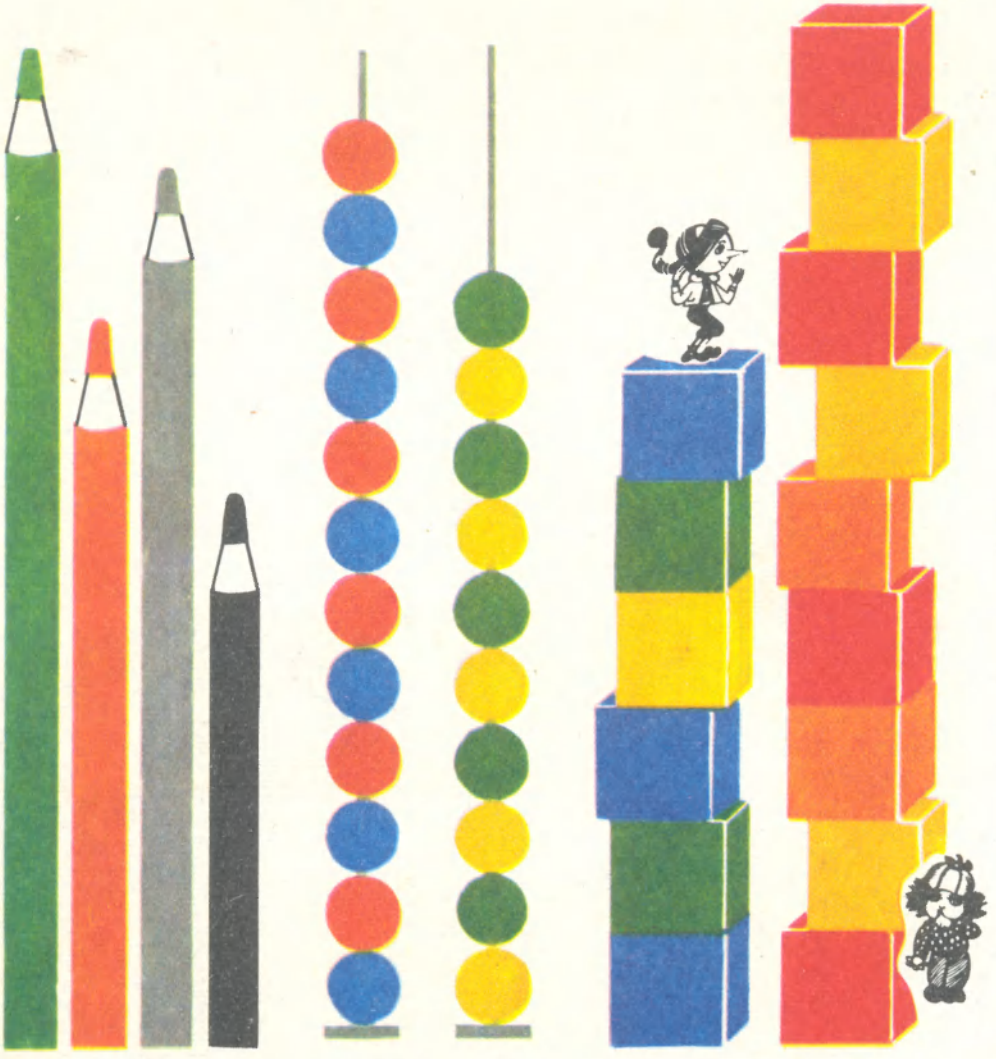
Then put it to the black segment. We see that the black is longer than the green. It turns out that you Dunno was not right either. The longest is the black segment."

! But which of these segments is the shortest? Now you draw some segments (not one under another, but at random). Take a pair of dividers and find the longest and the shortest among them.

Okay, now you can compare segments using dividers. The dividers will also help you to find which of any two objects is the longer (for example, the handle of a refrigerator or the handle of a bookcase).

To do this you won't always need a pair of dividers. In order to find which of the two pencils is the longer, you only need to

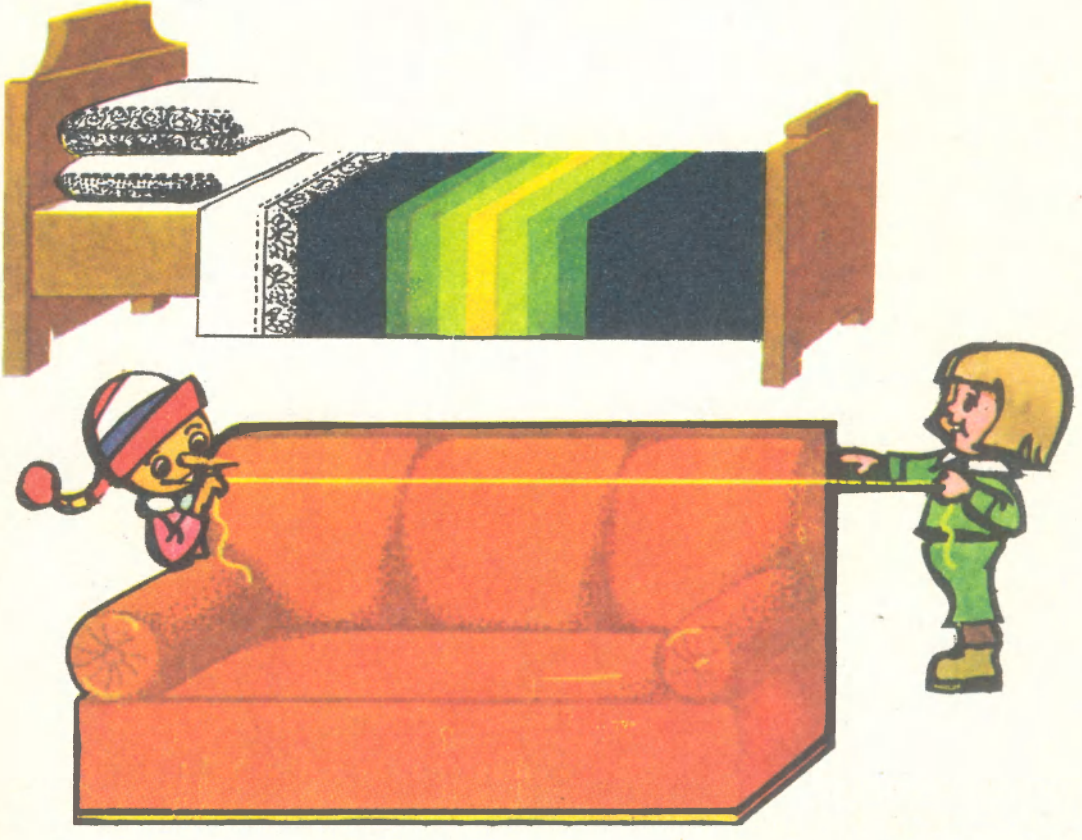


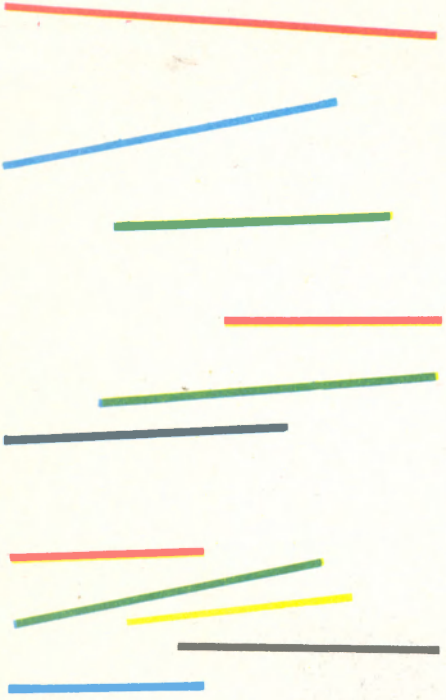


put them next to each other. You can compare sticks, toys and other objects in the same way.

But suppose you want to compare a sofa and a bed to find which is the longer? Dividers will be of no help here: they are too small. Also it's difficult to put a sofa and a bed next to

each other because nobody is going to move furniture just for this purpose. What's to be done then?
Gadgit and Pinocchio twigged. Look at the picture and tell how they found which was the longer of the two pieces of furniture.





WORKSHEET

1

Use dividers to compare these segments. Find the longest and the shortest.

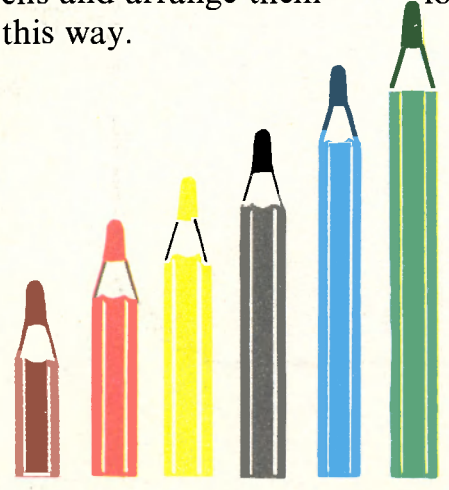
2

Are there segments of equal length among these?

And among these?

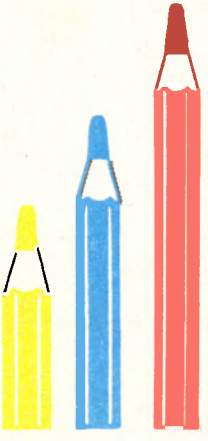
3

Peter arranged his pencils by length. Now you take your coloured pencils and arrange them this way.



4

.Ann's yellow pencil is shorter than her blue pencil and her blue one is shorter than her red one. Which is longer: the yellow or red?



5

Nick is taller than John but shorter than Dick. Who is taller, John or Dick?

6

Dol and Babs have the same height. Babs is taller than Marry, and Alice is taller than Dolly.
Who is taller, Alice or Marry?

7

Sam is taller than Mike, Jack is shorter than Bob, but taller than Sam. The boys stood in file according to their height, the highest standing the first. Who is standing next to whom?

8

Look at the objects in your home: tables, chairs, bookcases, stools, window-sills... Find which is the longer, the window-sill in the drawing-room or the one in the kitchen; which is the wider, the bookcase or the closet; which is the higher, a stool or the seat of a chair. Compare some other objects too.

"I want to listen to the fairy tale," said Dunno. "Pencil, will you go on with the story?"
"If you like," said Pencil. "Do you remember where we have stopped?"
"Yes, I do. Point asked the scissors to cut out several segments of the straight line, and he did so. Compasses and the ruler connected the remaining rays. And everybody saw that the straight line was safe and sound."
"Well, listen to some more."

POINT'S
Travels in Geometry Land

Point began to praise Compasses because he had connected the rays into a straight line:
"What a good pair of Compasses you are."
"The merit is not all mine," said Compasses. "Don't forget the ruler."
"But couldn't you alone connect the rays?"
"Of course, I could. But it might so happen that you wouldn't get a straight line."
"Why?" Point was surprised.
"We'll now see."

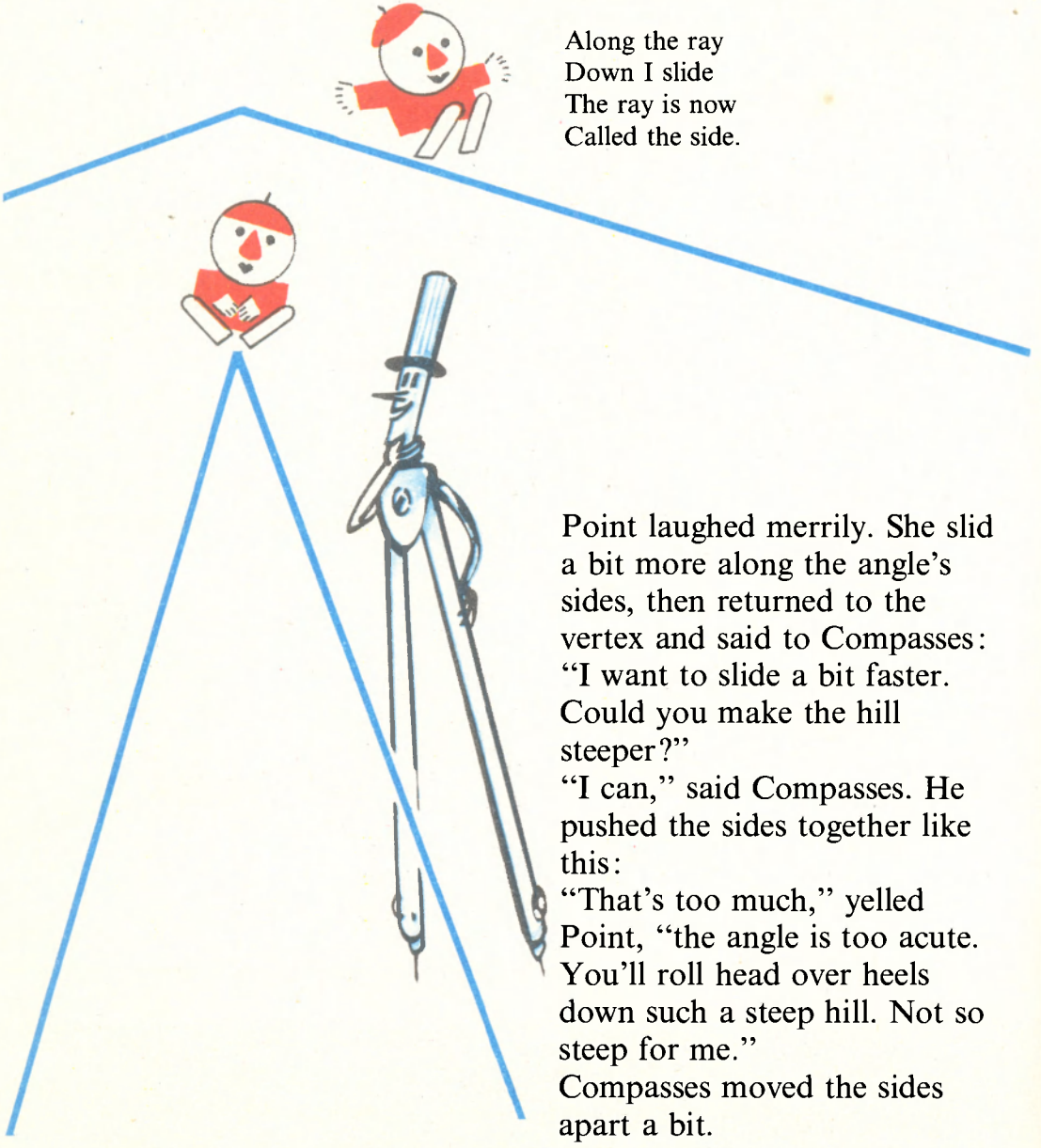
The scissors again cut the straight line into two rays.

Compasses drew the rays together and connected the ends. Here is what he got:
"Yes," said Point, "it is not a straight line. You cannot go straight here, you have to turn. What's this? What do they call this bit?"
"This is an angle," said Compasses.

"Angle... angle..." Point repeated the new word several times. "Compasses, what do they call the place where the rays connect?"

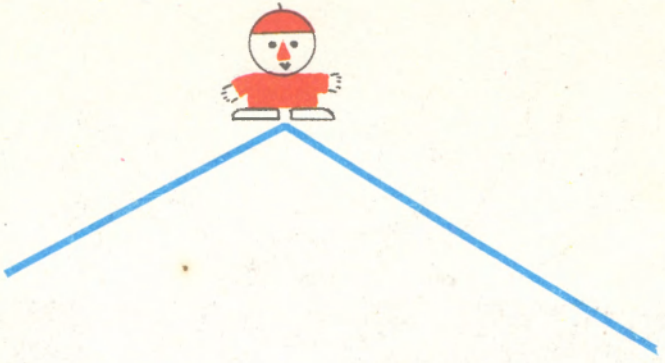
"The vertex of an angle. Now you, Point, stand on the vertex, the rays that emerge from you are called the sides of the angle."
"Oh, Compasses, wait, wait a little. So many new words. An angle, the vertex of an angle, the sides of an angle... How can I possibly remember all these?"

"Easy. Slide down the side of the angle several times like down a hill and you'll remember."
Point liked the advice. So she slid down one side and then down the other. She slid down and sang:



Along the ray
Down I slide
The ray is now
Called the side.

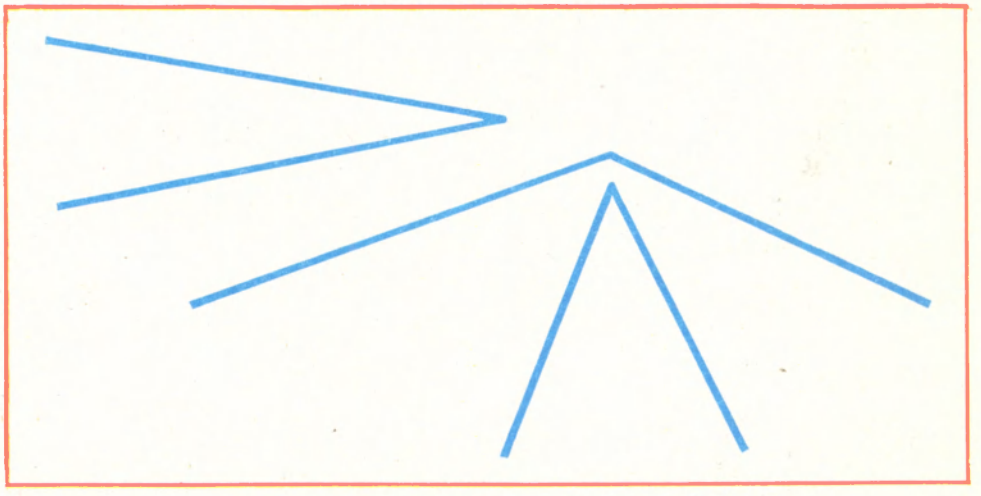
Point laughed merrily. She slid a bit more along the angle's sides, then returned to the vertex and said to Compasses: "I want to slide a bit faster. Could you make the hill steeper?"
"I can," said Compasses. He pushed the sides together like this:
"That's too much," yelled Point, "the angle is too acute. You'll roll head over heels down such a steep hill. Not so steep for me."
Compasses moved the sides apart a bit.



"It's okay now," said Point, "only I don't want to slide any more. You know, Compasses, what I'm thinking now: if we connect the ends of segments, and not the ends of rays, will we have an angle?"

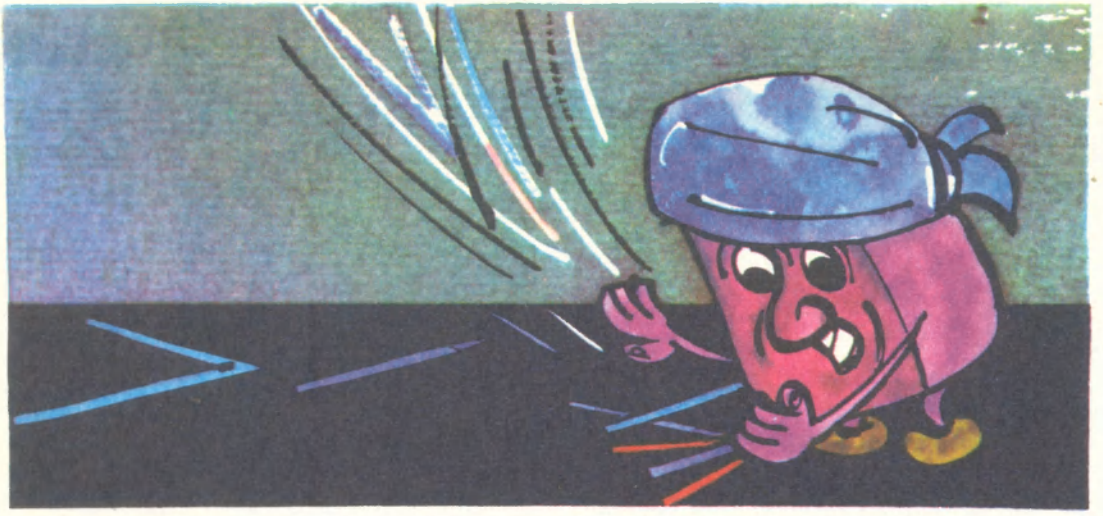
"Yes, rather..." said Compasses after some thought, "it can also be called an angle."

"Well, let's see," said Point. She remembered that scissors had cut many segments out of the straight line and called them in. The segments were glad that Point had not forgotten about them, and they came running and formed up in pairs... Presto! Each pair made an angle. Like these



"Look, Compasses, look," Point cried out merrily, "how many different angles! And the last one is just like you."

Compasses was just about to answer, when out of the blue came the wicked Rubber-Robber. He ran up to the first angle,



rub-rub-rub and erased it away. He pounced on the second one, rub-rub-rub, and erased the second angle away. And the third angle was ruined by the merciless Rubber-Robber. It would have turned out badly for Point too, but she managed to hide behind the back of Compasses. She had hardly gathered her wits, when Rubber-Robber vanished into thin air.

Point cried bitterly. She had just learned all about angles and wanted to look at them some more and now there weren't any angles any more. Little Point cried and Compasses consoled her.

"Don't cry, Point, don't grieve. We'll construct lots more new angles, both from rays and segments. That wicked robber will get into hot water soon. We'll catch him, punish him and make him do work."

The Happy gang listened silently. Gadgit

was sitting very serious, while Pinocchio was frowning, and Dunno was rubbing his eyes with his fist and even sobbed several times. Everyone was sorry for Point.

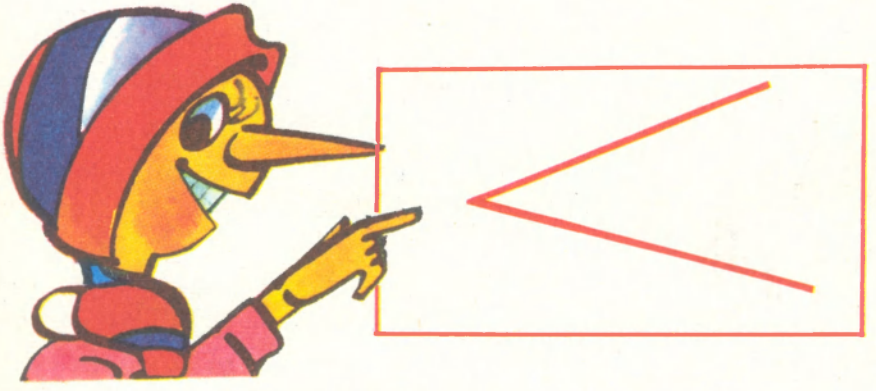
"Why looking so dejected?" Pencil addressed his friends. "Don't be so sad. It's just a fairy tale. And in fairy tales everything has a happy ending. You've heard what Compasses said. Just trust them to find Rubber-Robber. They'll teach him a lesson and will not allow him to do evil things any more. So never say die! We'd better remember what Compasses told and showed Point. Pinocchio, tell us."

? Do you remember what Point learned from Compasses?

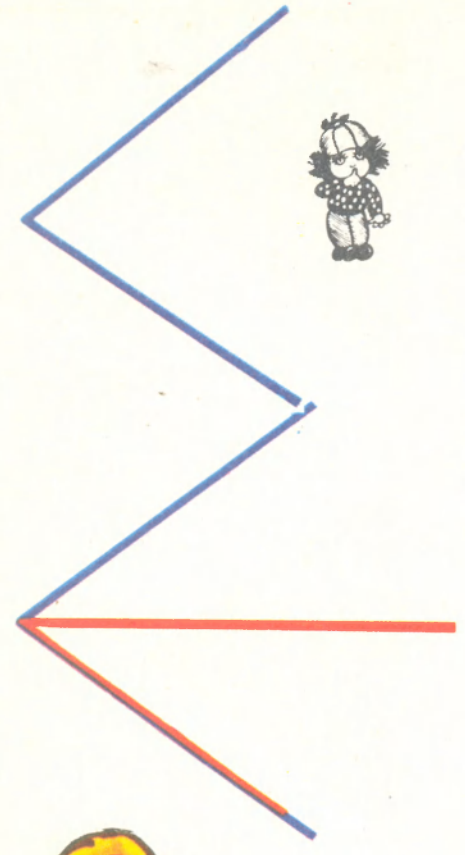
"I'd rather draw," said Pinocchio. "There isn't actually much to say. Here is an angle. Point learned what an angle is."

"And the vertex of an angle, you forgot the vertex," said Dunno.

"Nothing forgot I! This is the vertex and these are the sides of the angle," pointed Pinocchio.



! Now you point out the vertex and sides of the angle that Pinocchio drew. Draw some different angles. Show the vertex and sides of each of the angles. Count the angles you've drawn.



"I've also drawn an angle," said Dunno. "Look."

"By the way, your angle is a bit larger than Pinocchio's," said Gadgit.

"How is it: one angle larger than another?" asked Dunno.

"I just see it, but I cannot explain why," said Gadgit. Pinocchio was surprised:

"Is it really possible to compare angles?"

"Yes, it is," answered Pencil. "Suppose your blue angle, Dunno, and your red angle, Pinocchio, are made of coloured wires. You can put the two angles on the table one on top of the other, so that the vertices coincide and one side of the red angle goes along the side of the blue angle. The other side of the red angle goes inside the blue angle. That means that the red angle is smaller than the blue one, and the blue is larger than the red. Is that clear?"

"No, I don't understand," said Dunno.

Pencil reassured Dunno:

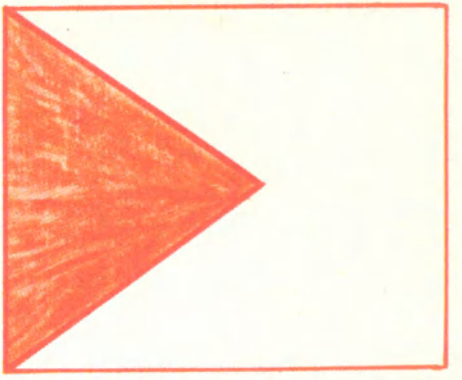
"Never mind, I'll explain it to you another way, and then everything will be clear. Just look, I draw an angle on a sheet of paper. In order that it could be seen better, I paint it inside. Now you draw an angle on

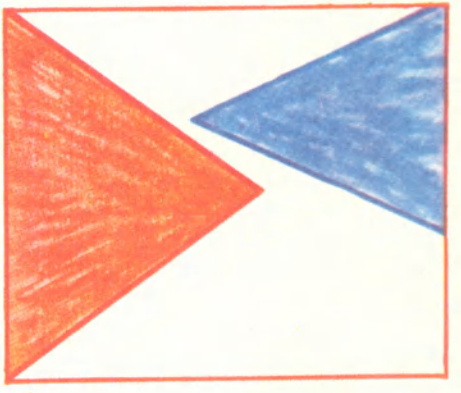
the same sheet and paint inside it. Now take the scissors and cut out what we've drawn."

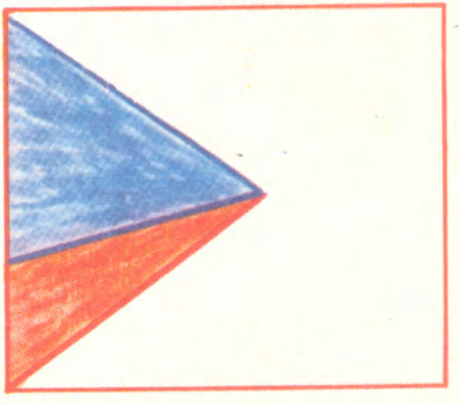
"Aha, I understand," Dunno cried out gladly, "I put the angles one upon the other and see which is the larger."

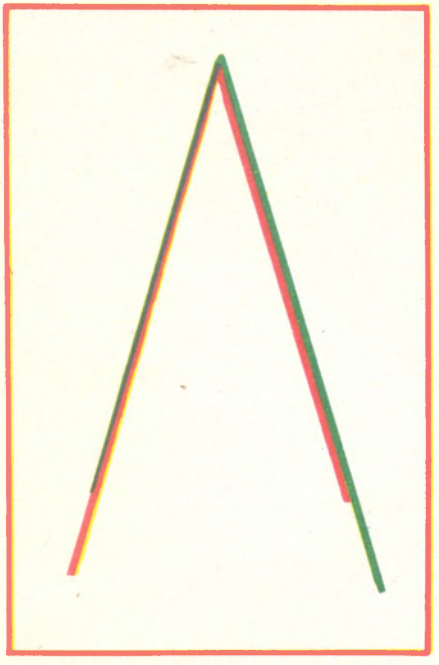
"Right! Only don't forget that the vertices must coincide," said Pencil.

"The red angle is the larger," Dunno showed his friends the result of his effort.









Pinocchio and Gadgit each made an angle of some wire and set out to compare them. Which is the larger? They put the angles together with the vertices coinciding. And it so happened that the sides of the angles coincided too. Look:
"Pinocchio and I have made equal angles," said Gadgit.
"Right!" nodded Pencil. "If the sides of two angles coincide the angles are equal."

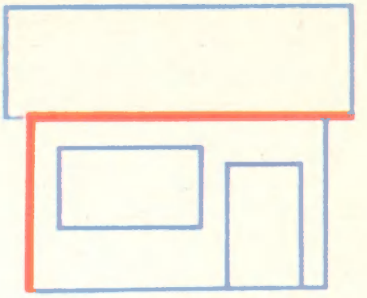
Now you make two angles from a piece of wire and compare them. Take a sheet of paper, draw two angles and paint them inside with different colours. Then cut them out and compare them.

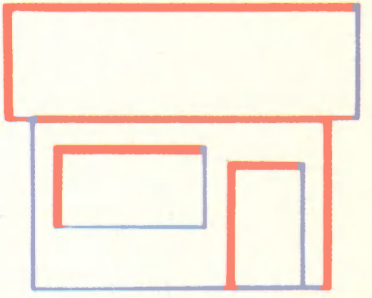
Pencil, Gadgit and Dunno drew angles, painted them with different colours, cut them out and compared them. They soon accumulated many bits of coloured paper. Gadgit thought of gluing them onto a long piece of string, and so they got a beautiful garland.
"We might leave this beautiful garland for a holiday," said Dunno.
Pinocchio has been seating idle all this time.

We might leave this beautiful garland for a holiday

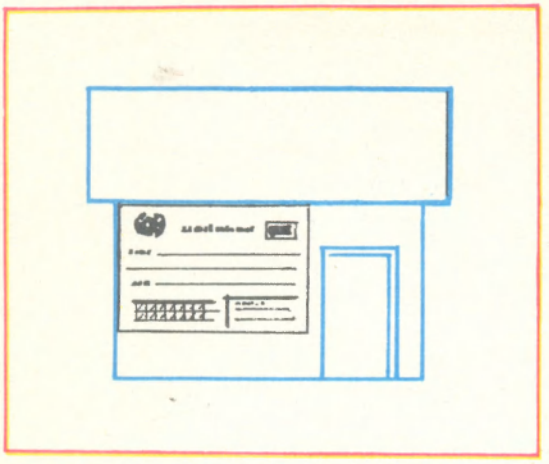
"I don't want to concern myself with angles any more," he muttered. "We've been drawing angles, and cutting, and comparing, but besides the garland we have invented nothing. What have we learned them for ? Who needs them?"
"Who?" cried out Pencil. "Why everybody: workmen, engineers, builders."
"Architects," continued Gadgit. "I've got an architect friend, Mr Skyscraper. He told me."
"What is an architect? Someone who builds houses?" asked Dunno.
"No, builders build houses. An architect draws the house on a sheet of paper and the builders build the house from the drawings. Well, let's visit Mr Skyscraper and see how he draws a house on paper. Pinocchio, you will see how many different angles there are in the drawing."
Mr Skyscraper was pleased to meet them.
"Well look, my friends, architects must show on their drawings everything that will be constructed by the builders: walls, roofs, doors, windows, and so on."
"But where are angles in this drawing? I don't see any angles," said the impatient Pinocchio.
"Just look carefully. Here, for example, is a segment representing a wall's edge, and the segment representing the roof's edge. They form an angle. Here is another angle, and another... you see?"
"I see now. There are many angles here. But it seems to me that they are the same, aren't they?"
"Yes, in this drawing all the angles are the same. These are right angles."

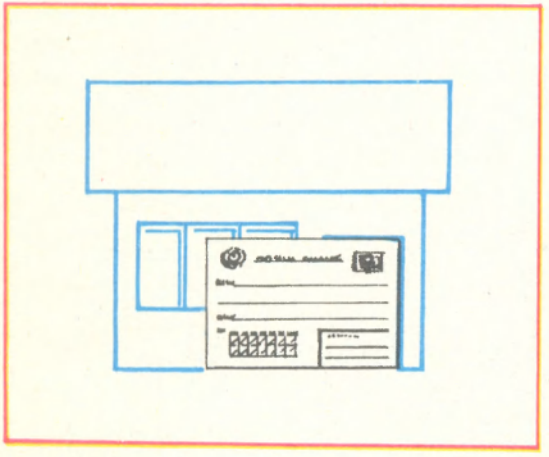
"Are they really the same!" cried Dunno suddenly. "They are absolutely different. The angle in the window is so small and the angle where the wall meets with the roof is large."

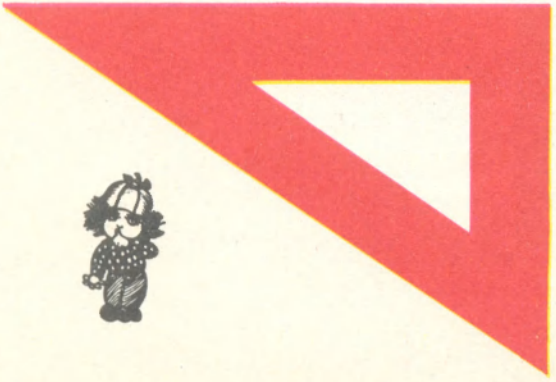




"Shame on you, Dunno. You forgot what equal angles mean," Pencil said reproachfully. "It doesn't matter if the angle's sides are long or short."
"You must put one angle upon the other. And if the sides of one angle will run along the sides of the other, then the angles are the same. Do you remember?"
"Oh, yes, I remember."







"So you see," said Mr Skyscraper, "all the angles in my drawing are really equal and all of them are right angles. Look, here is a postcard. Each angle of it is right. Apply it to the angles in the drawing."

Dunno did as he was told:

"Yes, the sides meet. That means that the wall and the roof form a right angle. Now I'll apply the card in another way. Look! Here too the sides run one along the other. It means that the angle at the window is right as well. I can put the card next to other windows, and to the door... You're right, all the angles in the drawing are right angles."

Gadgit entered into the conversation:

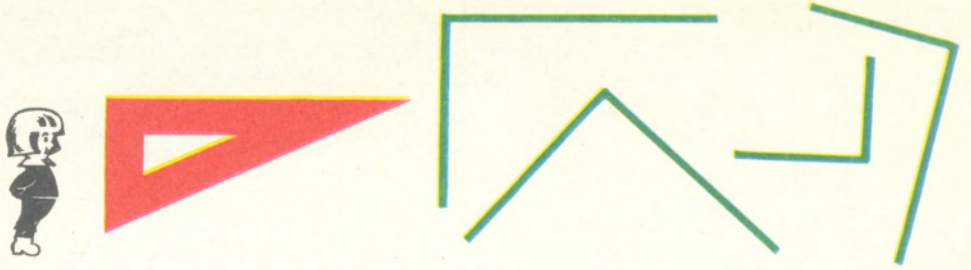
"Can we draw a right angle using the card? Put it on a sheet of paper and trace along two of the sides with a pencil."

"Of course, we can," said Mr Skyscraper, "but it's more convenient to use a set-square."

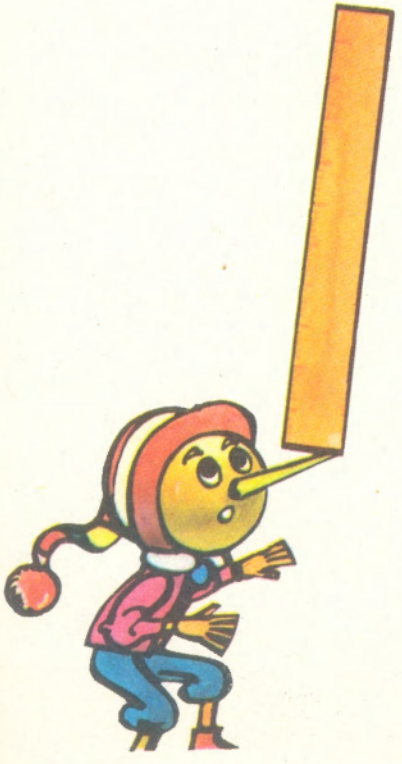
And he handed a set-square over to Gadgit.

"You see, the set-square has a right angle on it too."

Gadgit took the set-square and drew several right angles.



? Now you take a set-square and draw some right angles. Count the angles you have drawn. How many right angles did Gadgit draw?



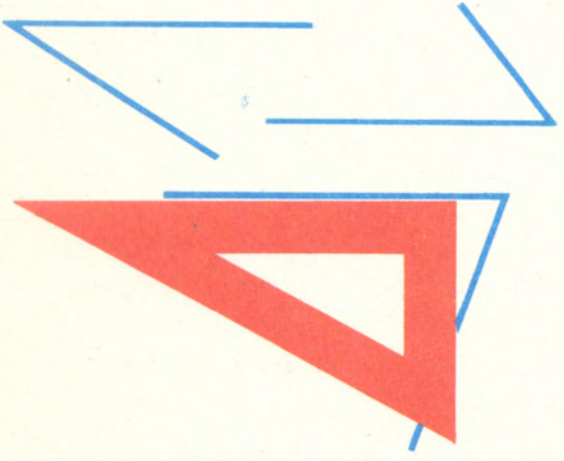
While Gadgit was drawing the right angles, Pinocchio found somewhere a large drawing ruler and tried hard to balance it on his nose, like a juggler. The ruler was disobedient and kept falling down from his nose and hitting Pinocchio hard on his hands and forehead. This didn't put Pinocchio off. He continued to have fun and even hummed a song that he had just composed:

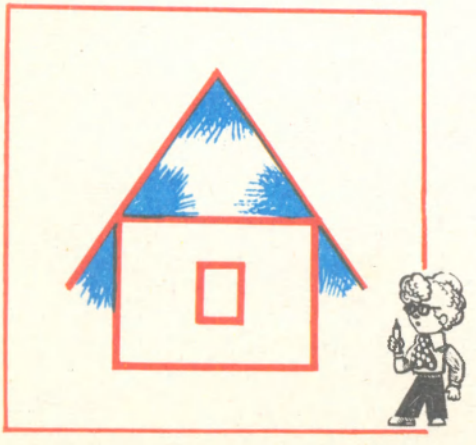
I'm a juggler and I shows
A ruler on my nose,
And everybody knows
That...

Now Mr Skyscraper had to take the ruler away from Pinocchio.

"You, little fidgit," he said, "have you learnt everything and understood everything?"

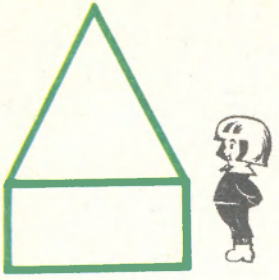
"Of course," Pinocchio answered promptly. "Architects only draw right angles."



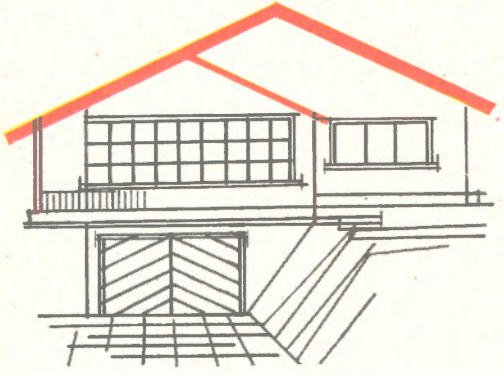


Mr Skyscraper laughed:
"You are jumping to conclusions again, Pinocchio. Look at this drawing: Is the angle of the roof a right angle?"
"No," drawled Pinocchio, "this angle is smaller than right."
"Right you are. This is an acute angle. Any angle that is smaller than a right angle is called acute. Look, I'll draw several acute angles. It's quite apparent that each of these angles is smaller than a right angle. But it is not always easy to determine whether an angle is acute by eye.
For example, is this angle acute or not? We'll have to check. So I take the set-square and use it this way.
Now you can see that the angle that I've just drawn is smaller than a right angle, so it's acute."
Then Dunno said:
"On the drawing of that cottage I see some more acute angles."
"Yes," said Pencil, "there are five acute angles on it all in all. Mr Skyscraper, may I mark them?"
"Yes, you may."

? Here is another drawing of a house. Point to the acute angles and all the right angles on it. Count the acute and right angles. How many angles are there in the drawing in all?

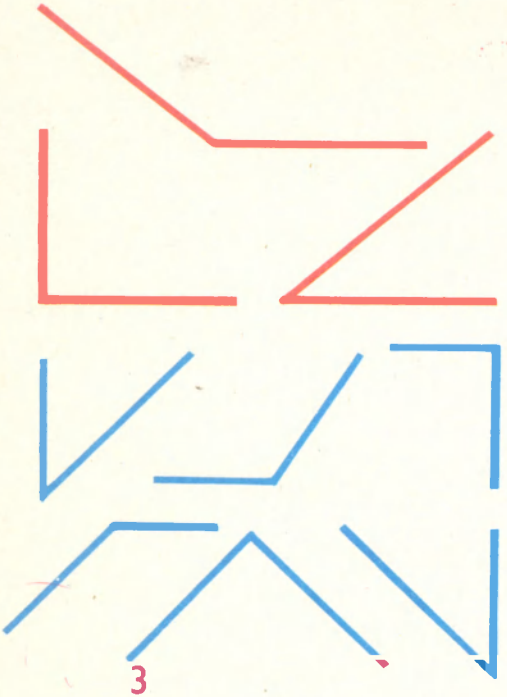


"Tell me please, Mr Skyscraper," Gadgit asked suddenly, "what about angles that are larger than a right angle, do they have a name too?"
"Of course," the architect smiled approvingly at Gadgit. "They are called obtuse. Look at this drawing:"



"The angle at the top, the one with the roof, is obtuse. You can see that it's larger than a right angle even without checking."
"Why does the roof of one house have an acute angle, and of the other an obtuse angle? Why do they build houses so differently?" asked Pinocchio.
Mr Skyscraper explained:
"If a roof has a very obtuse angle, then during the winter so much snow may accumulate on it that the roof will collapse. It means that in those climates where they have a lot of snow in winter a roof with an acute angle is better because the snow slides down. But if a house is in a warm climate, it is not necessary to make a roof with an acute angle. They even build houses with a flat roof there."
Mr Skyscraper has told the Happy gang lots of interesting things about how architects design houses, why they build different houses in different countries. And how architects need geometry.

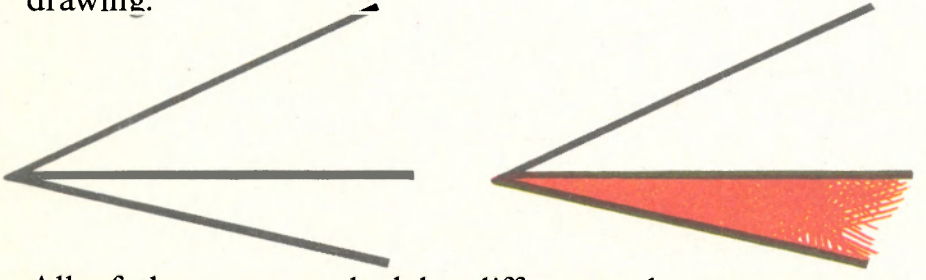
WORKSHEET



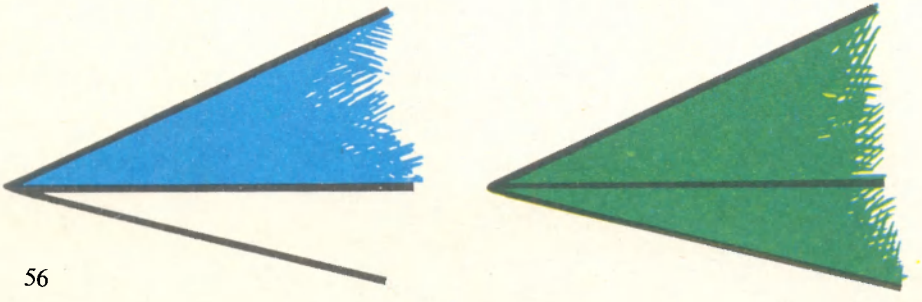
1
Which of these angles is right, which is smaller than a right angle, and which is larger than a right angle?

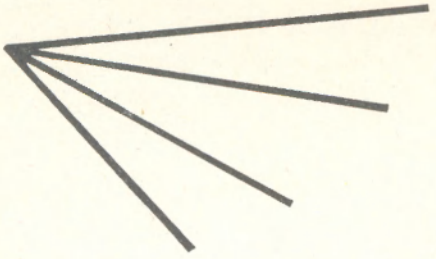
2
Take a set-square and check to see if there are any acute angles here.
And obtuse? And right? How many acute, obtuse and right angles are there?

3
There are three angles in this drawing.



All of them are marked by different colours.
But in this drawing there are six angles.





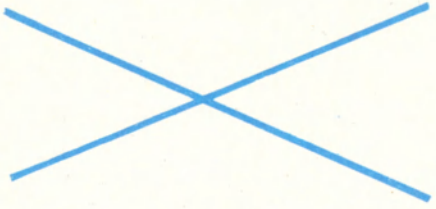
Find each of them and mark each by some colour.

4

Take a set-square and draw two equal acute angles.
Now draw two unequal obtuse angles.

5

Is it true that every acute angle is smaller than any obtuse angle?

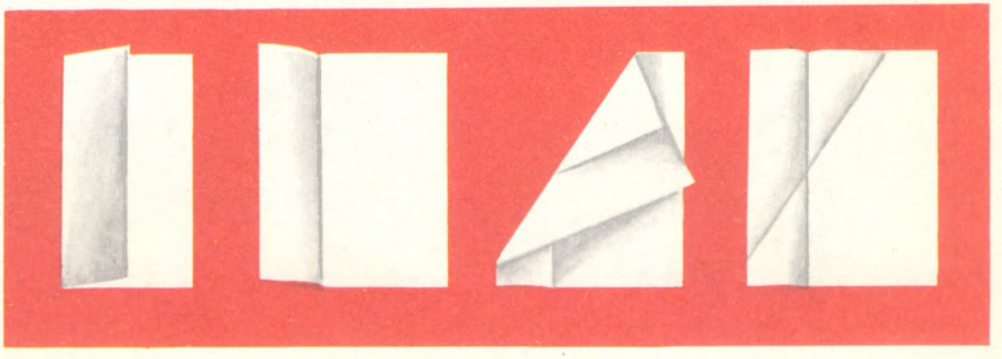


6

There are two acute and two obtuse angles in this drawing. Point to them.
Draw the same picture on a sheet of paper and colour the acute angles one way and the obtuse another.

7

Take a sheet of paper. Bend it and then straighten it.
Now you've got a straight line at the bend. Now bend it in another direction. Again straighten it.
Look at the angles that you've made without a pencil and ruler. Mark them with different colours. By bending a sheet of paper you can also obtain right angles. How?



8

Take two sticks. Put them together to make an angle between them. Make an angle out of a piece of wire. What angles have you got?

9

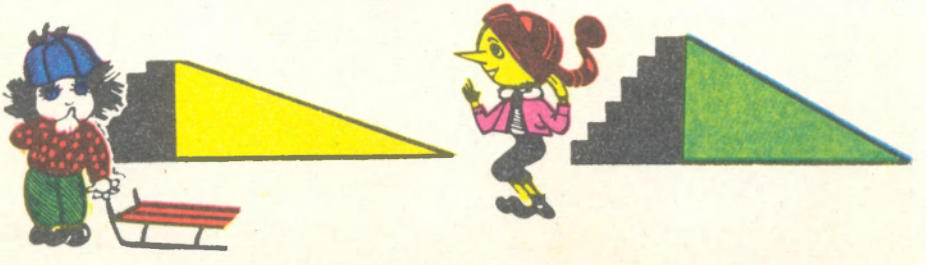
Make an acute angle using your sticks. Now shift them apart so that you get a right angle. What angle will you get if you continue to push the sticks apart?
Do the same with the angle of wire.

10

Arrange some of your pencils like this:
Which angle is the larger, the one between the blue pencils or the one between the red pencils?
How must you move the blue pencils for the blue angle to be larger than the red angle?

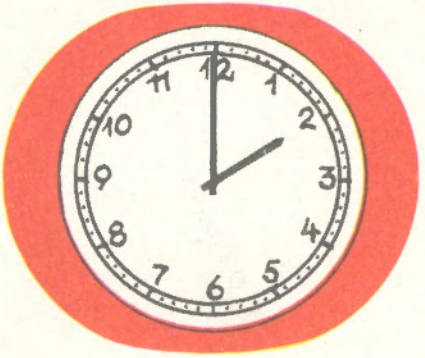
11

On a children's play-ground two hills have been made, one yellow and one green.
Look at the angles Dunno and Pinocchio are pointing at. The Happy gang are having a difference: Pinocchio says that the green hill's angle is larger, but Dunno says that the yellow hill's angle is larger. Who is right? Which hill is steeper?
Which hill would you slide down faster?



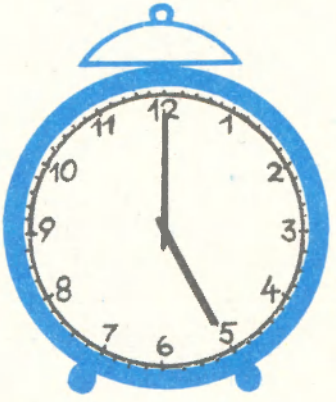
12

Two sticks were sharpened in a different way:
Which angle is acute, and which is obtuse?
Which stick would be easier to drive into the ground?

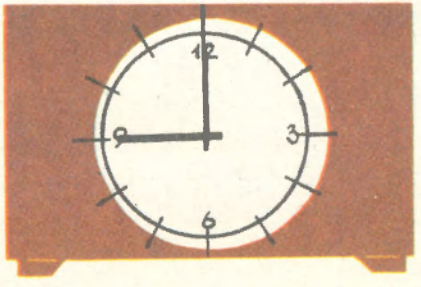


13

Look at the clock. The hands make an angle too.
The wall clock in the picture says it's two o'clock.
Which angle is between the hands? Will the angle become larger or smaller in five minutes' time?



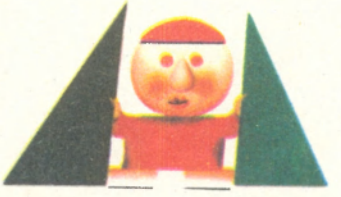
The alarm clock shows five o'clock. What is the angle between the hands? Will the angle become larger or smaller five minutes later?



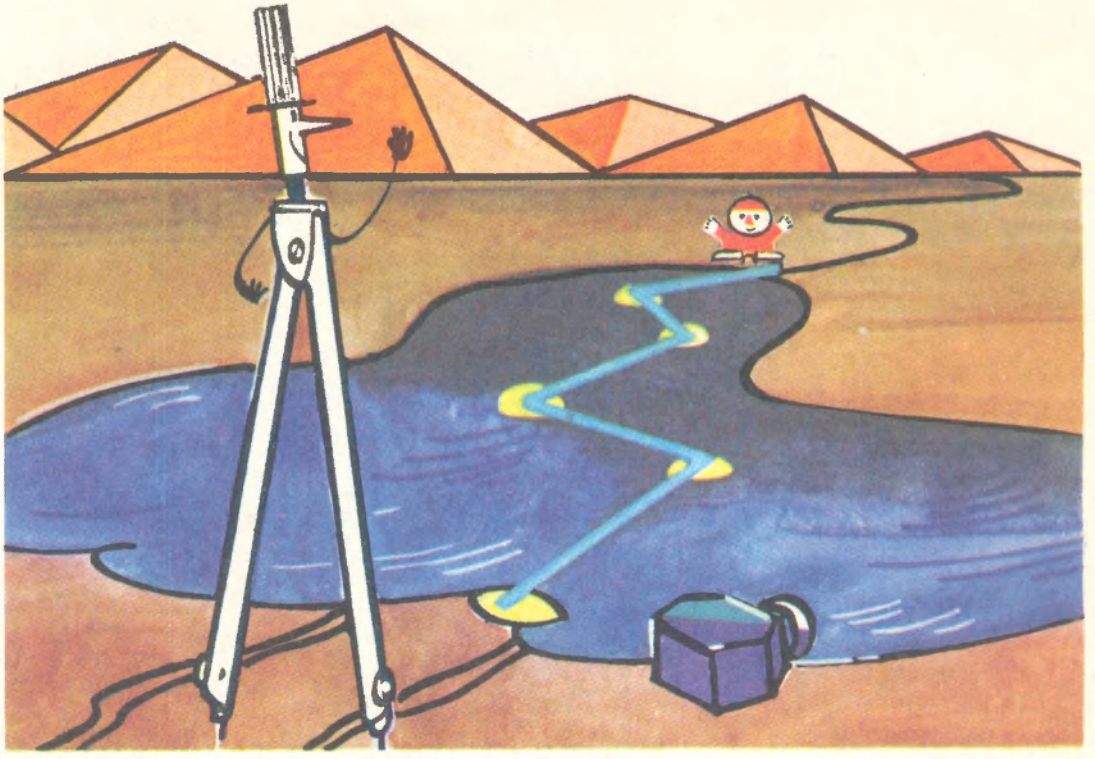
The desk clock indicates nine sharp. You can see that the hands make a right angle. Do you know some other times when the hands form a right angle?

When the Happy gang had tea at Pencil's again, Pinocchio said:
"I wonder if Point and her friends will find the wicked Rubber-Robber? He needs a good thrashing!"
"Yes, Pencil, it's been a long time since we listened to your fairy tale. Please tell us some more," Dunno pleaded, "I want to know what will happen to Point."
"And what else she will learn," added Gadgit.
"Well," said Pencil, "listen."

POINT'S
Travels in Geometry Land



...Little Point cried and Compasses consoled her: "Don't cry, little Point, don't be sad. We'll catch that Rubber-Robber and punish him and make him do some useful work."
So Point and Compasses set out on a journey. Compasses strode fast ahead. His legs were very long, but the poor little Point scuttled behind, hardly catching up with Compasses. Compasses saw that Point could not go at his pace, and so he put Point on his shoulders and walked even faster. He had been walking for an hour or so when suddenly he stopped: a huge ink sea barred their way. They could not go round it or skip over it. Rubber-Robber must have put it there to stop them.
"What's to be done," asked Point. "Will we really have to go back?"
"Why, no," said Compasses. "If we scratch our heads a bit we'll find a way out. You see the islands in the sea? Of course, I cannot reach them, but we can build a bridge!"
"But how?"
"Our friends, the segments. We'll call them to help!"



No sooner said than done. One segment spanned the gap between the shore and the nearest island. Another segment ran along him to the end, took hold of his hand and – bam! – hit the next island. A third segment ran along the first two, followed by a fourth, and a fifth and... Bam-bam-bam – the bridge was ready.

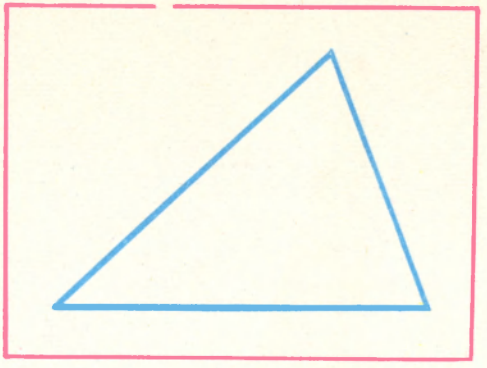
"Hurray!" cried Point, "what a good bridge! And what an interesting line we've got. Compasses, what do you call it? Of course, it isn't a straight line."

"This is a broken line."

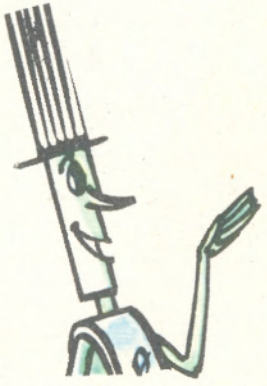
"Ha-Ha," laughed Point, "what a funny name, a broken line. Who broke it? So you can get a broken line out of segments."

"Yes," said Compasses. "We'll use this broken line to get to the other shore."

They crossed the sea and went on. On and on they went until suddenly they saw a town. They wanted to go a bit closer but a guard stood in the way and would not let them through.
"Why don't you let us pass?" Point asked in surprise.
"Because a road starts here that leads to our town," answered the guard, "and we only let through those who have already learned something about geometry and want to learn more."
"Then you must let me in. I know a lot about geometry."



"What do you know?"
"I know about straight lines, segments, rays, angles, and broken lines."
"Oh no, that's not very much! For example, do you know about triangles?"
"No, I don't."
"But do you want to learn about them?"
"Of course, I want to."



Compasses entered into the conversation. He called in three segments and they connected themselves at their ends thus:
"What is this?" Compasses asked Point.
"It isn't a broken line, is it?" cried out Point.
"True, but how many segments are there in it?"
"Three."
"And how many angles?"
"Let me count them. One, two... three. Three as well."
"Well, this is what is called a triangle. The segments in it are called the sides of the triangle, and the vertices are the vertices of the triangle."
"I see," nodded Point. Then she looked carefully at the guard and said:
"Now I understand why you asked me about triangles. You are all triangular."
"Of course," answered the guard. "All the inhabitants of our town are triangular. It's called the Town of Triangles."
"And now, will you let us into the Town of Triangles?"
"Yes. Pass through."
Point and Compasses entered the town. It was a very surprising town. Everything in it was triangular. The houses were triangu-

lar, windows and doors were triangular. Triangular flowers grew in the streets, and triangular apples and triangular pears hung on triangular trees in gardens.

Little Point was delighted with the town.

"Oh Compasses, just look how very interesting everything is here! How many triangles there are around and how different they are. Look, look! This one is so long and gaunt, it's so funny. And the one over there has become unfit, how it manages to stand on its legs!"

"Yes," muttered Compasses, "I've seen many triangles in my life, but I've never been to the Town of Triangles. It's really very interesting here."

Suddenly Point and Compasses saw something strange: a house stood before them but for some reason it wasn't triangular. It seemed that somebody had damaged it.

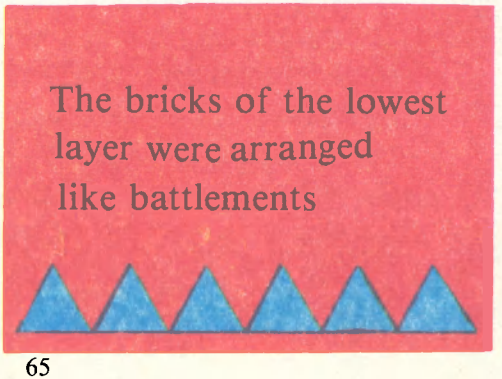
"Who could damage a house like that?" Point was outraged.

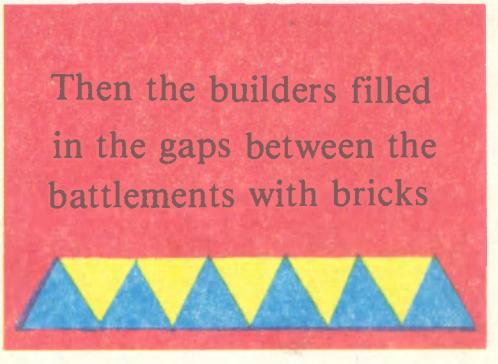
"It was the wicked Rubber-Robber," said a triangle that was passing by.

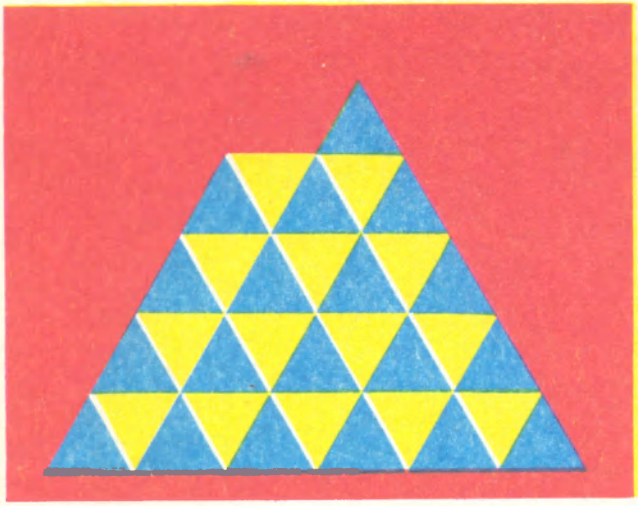
"Why! He's been here too?" exclaimed Compasses.

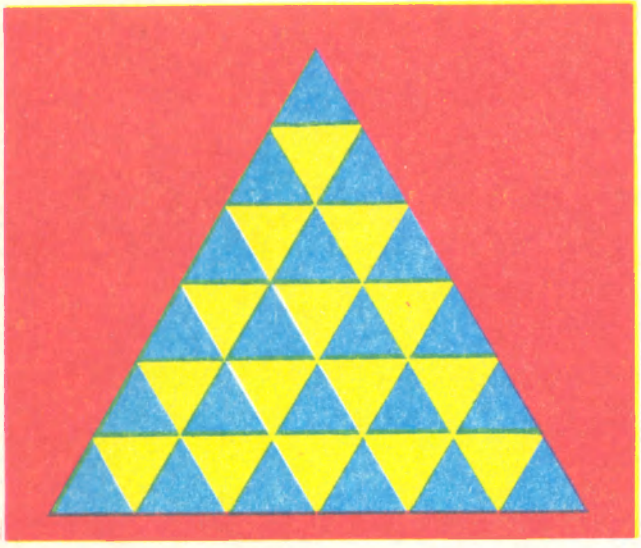
"Yes, yesterday he attacked our town, damaged several houses and trees, and even erased some of them completely. Our builders have much work now: everything must be repaired quickly."

Point and Compasses came closer to the damaged house and watched the triangle-builders erect a new wall of bricks (which, of course, were triangular).









Then they laid down a fresh layer of battlements, and again filled in the gaps... The wall rose higher and higher under their very eyes. The builders worked skillfully. When they saw that Point and Compasses were watching them, they waved at them, looked at one another and started singing all together:

We live in Town of Triangles
And know us you might.
We have three sides and have three angles,
As trivet are we right.

The work went faster with the song and the wall was ready in no time.

"As trivet are we right," Point repeated the last words of the song which she liked very much. Then she said:

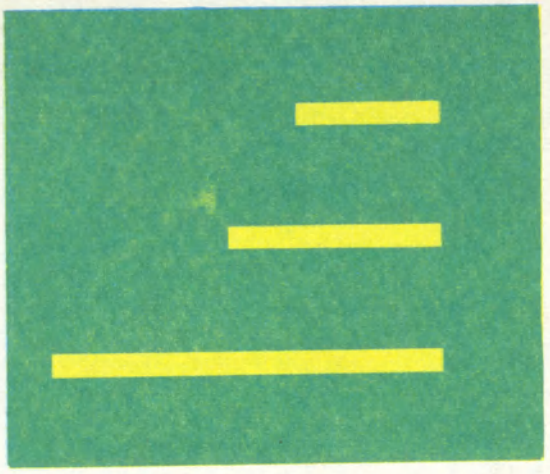
"Rubber-Robber has also hurt Compasses and me, he ruined lots of angles and nearly erased me. So we decided to catch him and punish him. We have been travelling a long time but we haven't yet found his hide-out."

"We don't know where he is either," said the triangles, "but it's time the villain was taught a lesson. Let's join forces. Will you have us as assistants?"
"Of course," said Compasses, "let's all walk together."
"No," said the triangles, "walking would take too long. After all, we could travel much faster."
"But how?" cried Compasses and Point in unison.
Here Pencil stopped to get his breath.
"Let's call it a day," he said, "we'll have some more next time."

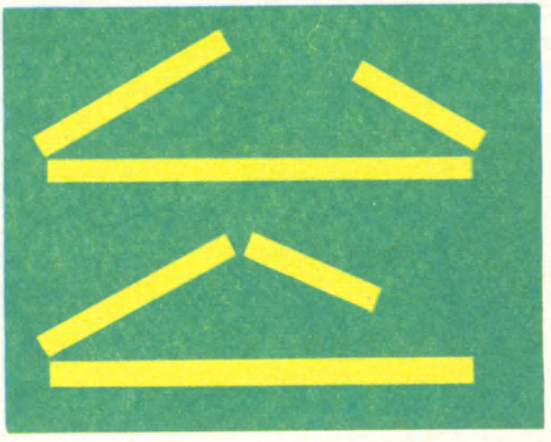
"I know how they are going to travel now," Pinocchio said confidently. "They are going to ride in a car, eh? That's right, isn't it, Pencil?"
"I don't know... Maybe. You are in a hurry again, Pinocchio. Let's wait until next time."
"What are we going to do now?" asked Dunno.

"What do you think?" Gadgit was surprised, "we could draw triangles or put them together out of sticks..."
"So you think it's difficult to make a triangle of sticks..." said Pinocchio contemptuously. "Just take three sticks, connect their ends in pairs, and – presto – you've got a triangle."
Pencil smiled:
"So you think that any three sticks will do to make a triangle, do you?"
Pinocchio took three sticks from the table and put together a triangle.

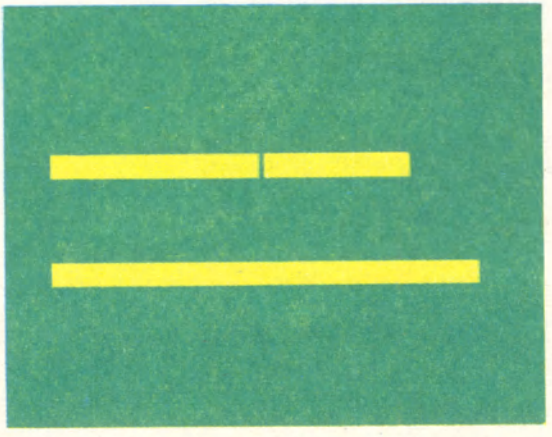
? And what do you think: are any three sticks suitable for making a triangle?



"Okay," said Pencil, "now take these three sticks and make a triangle of them."



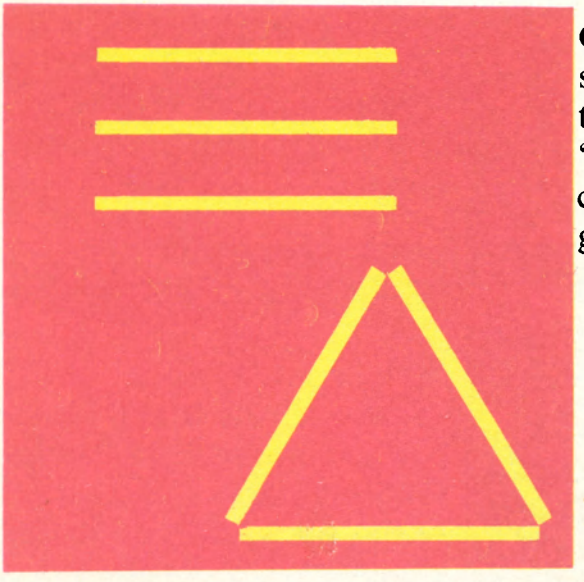
"As you please," Pinocchio started confidently but... could say nothing more: try as he could he got no triangle out of the sticks. Pinocchio tried hard putting the ends of the sticks together in different ways, first like this and then like this but each time the ends of two of the sticks would not come together.



"Well?" grinned Pencil.
"Alright," conceded Pinocchio.
"Oh," sighed Dunno.
"I knew it!" exclaimed Gadgit.
And the four laughed together.
Pinocchio said:
"I was wrong. There's no way of making a triangle here."
"Of course there's no way," said Gadgit. "The two smaller sticks together are shorter than the largest one. You see?"

"So, my friends, remember," said Pencil, "if you want to make a triangle out of three sticks, any two sticks must be longer than the third one."
"That means that in any triangle any two sides are together longer than the third one. Is that right?" asked Gadgit.
"Right."

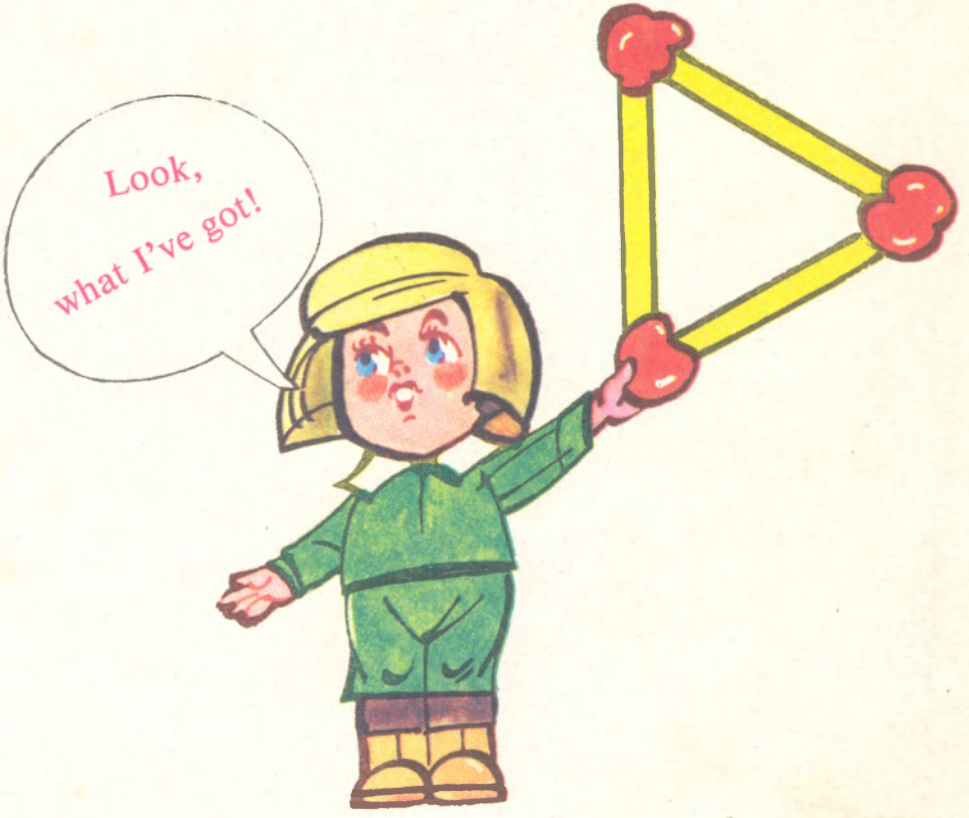
! Make a triangle out of three sticks. Check if any two of the sticks are together longer than the third one. Choose some three sticks that cannot be used to make a triangle. Explain why you cannot make a triangle of them.



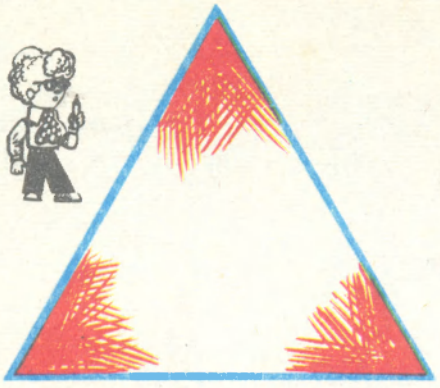
Gadgit took three identical sticks like these and made a triangle of them.
"Out of three equal sticks you can always put together a triangle," he said.

"Right you are," nodded Pencil. "It is said about this triangle that all its sides are equal. Therefore it's called equilateral, where 'lateral' is the Latin for side."

While Pencil was talking, Gadgit took some Plasticine and stuck the sticks of his equilateral triangle together.
"Look," Gadgit showed it to his friends, "I stuck some Plasticine at all the vertices of the triangle. Now you can handle it without its going to pieces."



Now you take three equal sticks and make an equilateral triangle. Take some Plasticine and use it to stick the connections together just like Gadgit did. Your triangle won't fall apart if you hold it in your hands.



"Notice," said Pencil, "that in an equilateral triangle all the angles are equal too. And each of them is acute."

"I thought of something," Dunno suddenly jumped up. "A triangle with a right angle... Can it be so?"

"Of course," said Pencil. "It is very easy to draw one."

"How?"

"First of all draw a right angle."

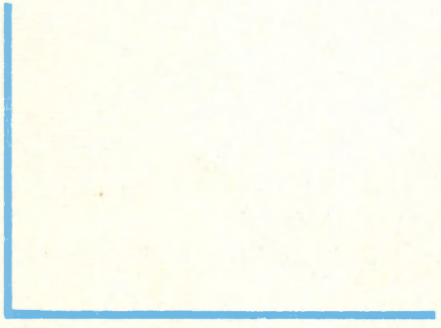
Dunno took the set-square and leaned over a sheet of paper. The right angle was ready in no time.

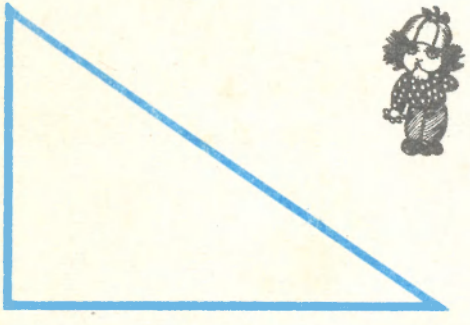
"Now connect the ends of the segments."

"Well, I got it! Here is a triangle with a right angle. What do they call it?"

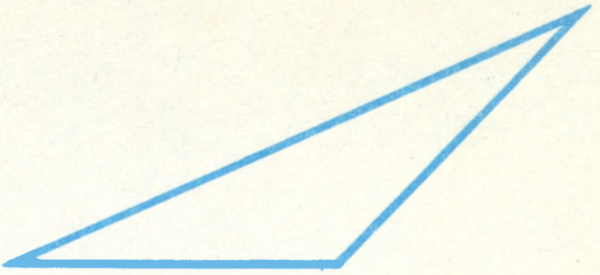
"A right-angled triangle."

Dunno was very pleased. He drew some more right-angled triangles.



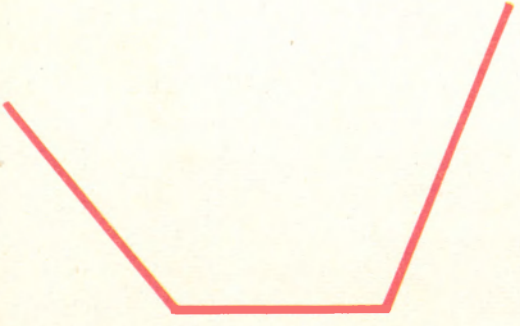


Now you draw some right-angled triangles.

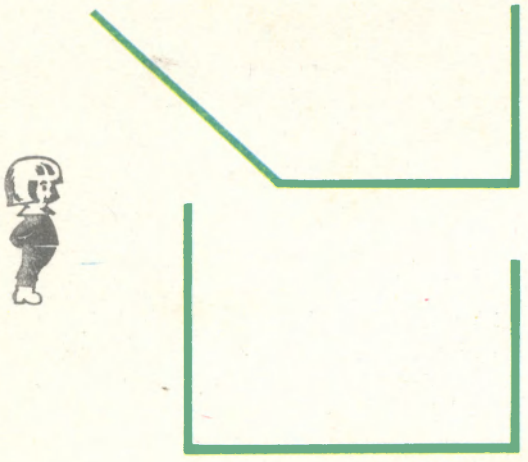


Dunno was busy with his sheet of paper for some time and then showed his drawing to his friends.
"And this is a triangle with an obtuse angle. Pencil, what is it called?"
Pinocchio smiled:
"Dunno, you are just like the little Point in the fairy tale. She was always asking 'What is it called?' Clearly, a triangle with an obtuse angle is called an obtuse-angled triangle."
Dunno took offence because Pinocchio had compared him to the little Point.
"I see it's not allowed to ask questions, eh" he said. "If you are so clever, Pinocchio, you tell me what a triangle with two obtuse angles is called."

Think about it: is there a triangle with two obtuse angles? Let's see how Pinocchio answers Dunno.



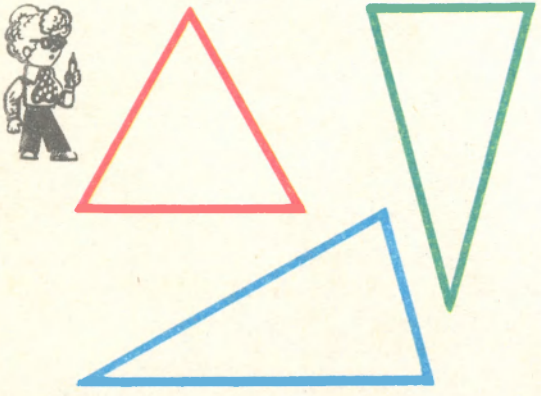
Pinocchio twigged that there is no way of constructing a triangle with two obtuse angles. Otherwise two of the three segments would separate like this: and could never have their ends connected.

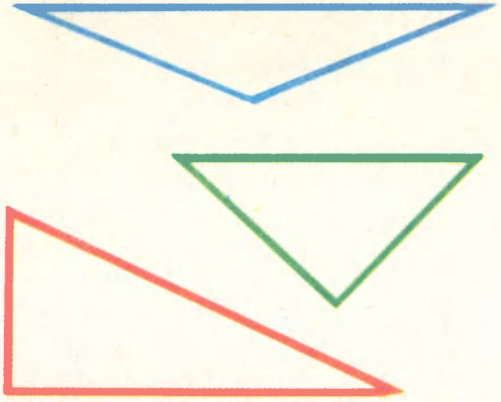


"There are no such triangles," said Pinocchio.
"And also there is no way of having a triangle with one obtuse angle and another right angle," added Gadgit.
"Nor can you get two right angles in a triangle."

Explain why there can never be a triangle with two right angles and a triangle in which one angle is obtuse and another is right.

Pencil listened attentively to the discussion.
"We've thus found out what sort of angles can make up a triangle," he said. "We now know that two of the three angles in a triangle must be acute. As for the third angle, it may be either acute, right, or obtuse. And the name of the triangle depends on what this angle is. If it's acute, the triangle is called an acute-angled triangle, if it's right, the triangle is called a right-angled triangle, and if it's obtuse, the triangle is obtuse-angled. Have you remembered?"

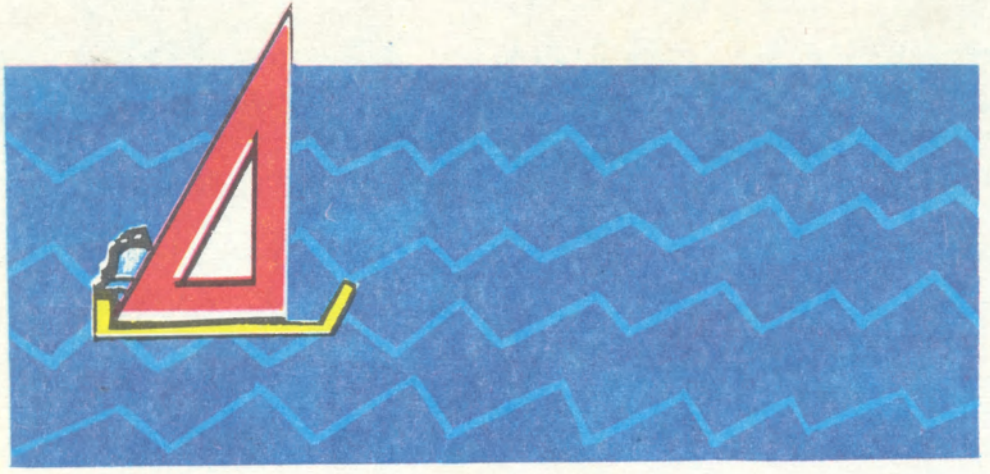




"Remembered," answered Dunno for his friends, "but I'm tired of learning. Let's go for a walk."

The Happy gang ran out of doors and began playing games. Each time somebody had to be It, Pinocchio used a new rhyme. He had just invented it when they were listening to the fairy tale.

Einy meeny miny mo
Once a point fell in a pit
Then a segment helped her so
Now she says that you are It.



The friends played into the evening. At night, when everybody was sleeping, Dunno had a dream. He dreamed that he was a famous traveller and that he was travelling in Geometry Land. He took a broken line made up of three segments and made a boat. Then out of another broken line that was made of many segments he made a sea and set out on a journey in his boat. Then he made for the mountains. Their summits rose up high into the sky with their acute angles. But Dunno easily climbed up the highest and steepest mountain.

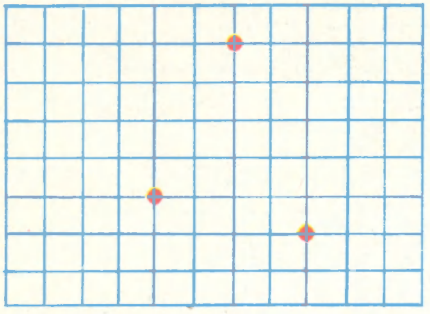
Then for some reason the mountains turned into triangles. They crowded around Dunno and began to ask him "What am I called?... What sort of triangle am I?... What am I?... And I?..." Triangles flashed before his eyes. He couldn't work out whom he should answer. Dunno was completely lost, and he just stood dumbfounded. Then one of the triangles stood forward and shouted out loudly so that the others could hear, "Stop! Don't ask him. Obviously, he doesn't know anything. We'll have to show him." Then something very strange happened for the triangle began to change its

shape. First it was obtuse-angled, then suddenly right-angled... then it turned into an acute-angled triangle. Dunno watched in surprise as the triangle transformed itself, while it merrily recited:

Everyone should know me,
They need not be too bright.
My angles' names are three:
Obtuse, acute, and right.

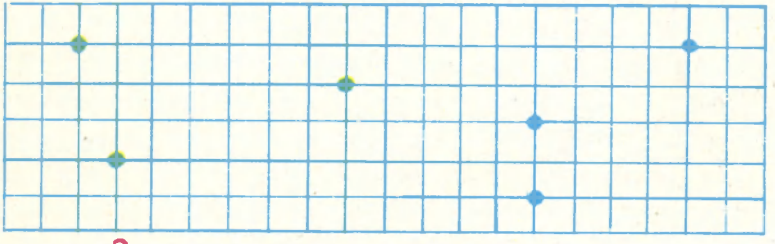
'I do so know all about triangles!" Dunno wanted to yell and... woke up.

WORKSHEET



1

Put three points on a sheet of squared paper like this: If we connect the three points with segments, they'll become the vertices of a triangle. Connect them. What sort of triangle have you got? What sort of triangle do these vertices belong to?

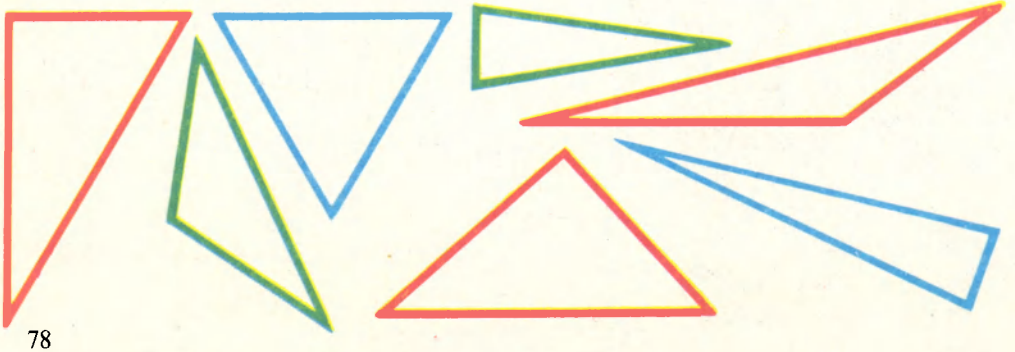


2

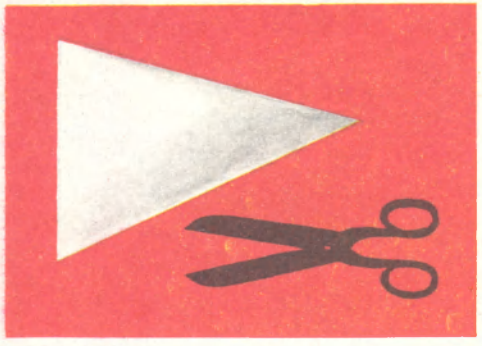
Rearrange the three points on the sheet of squared paper so that they become the vertices of an acute-angled triangle. Now place the points so that they become vertices of a right-angled triangle; of obtuse-angled triangle.

3

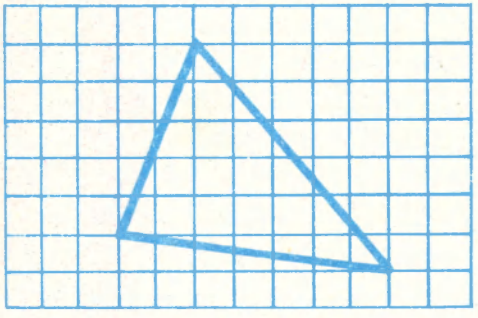
Find among these triangles all the acute-, right-, and obtuse-angled triangles.



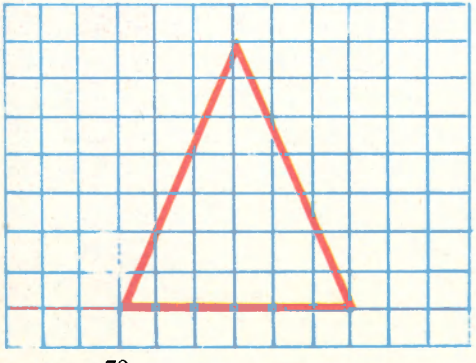
4
Draw an acute-, right-, and obtuse-angled triangle on a blank sheet of paper. Paint each of them with a different colour and cut them out.



5
Cut a triangle out of a piece of paper. Think about how it's possible to cut the triangle along a straight line to obtain two triangles.



6
Draw the same triangle on a piece of squared paper. Point out vertices of the triangle. Find the shortest and the longest of its sides.

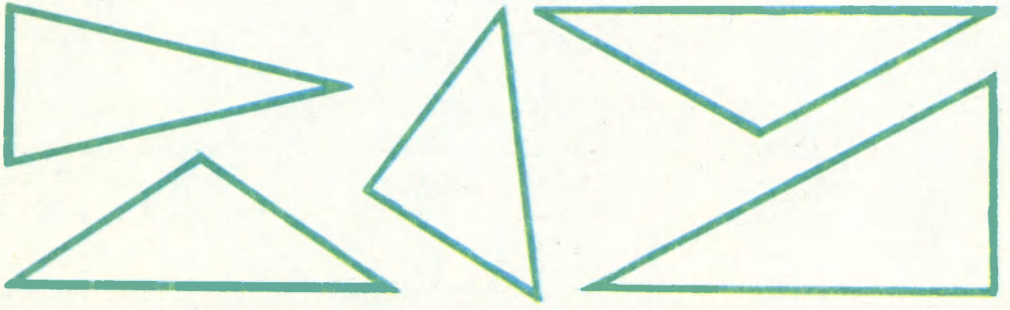


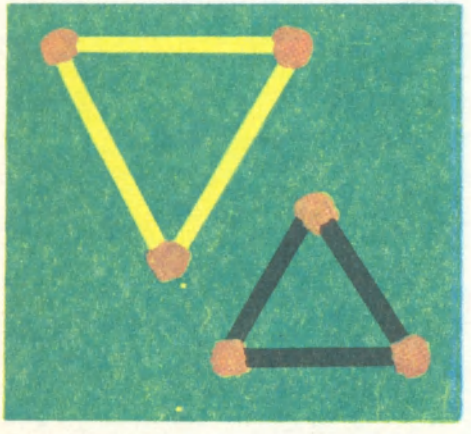
7
In this triangle two sides have the same length. Point to them. A triangle with two equal sides is called an isosceles triangle.
Here are some more isosceles triangles.

Point to the equal sides in each of them.

8

Are there any isosceles triangles among these triangles? How many?





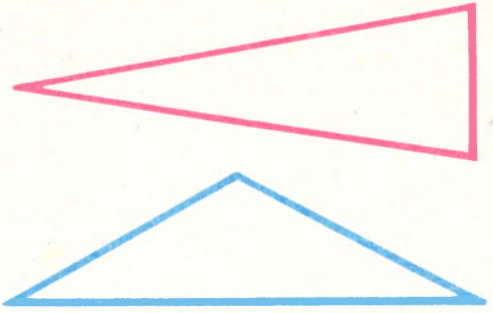
9

Using some sticks and Plasticine make two equilateral triangles. By putting these triangles one upon the other you'll see that all the angles are the same.

10

All the sides in the equilateral triangle are equal, so it has even more than two equal sides. Therefore, an equilateral triangle can also be said to be an isosceles one. Is the opposite true, that is, can you say about each isosceles triangle that it's equilateral?

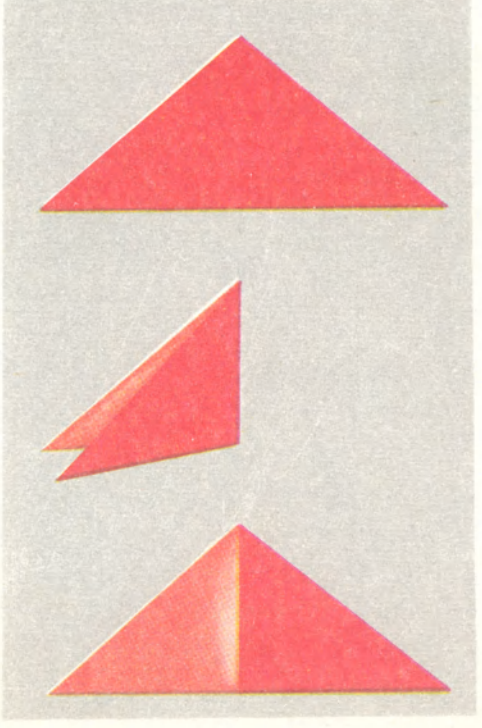
11
Draw an isosceles triangle that isn't equilateral.



12
This is an acute-angled isosceles triangle.

This is an obtuse-angled isosceles triangle.

Draw a right-angled isosceles triangle (the best way is to use a sheet of squared paper).

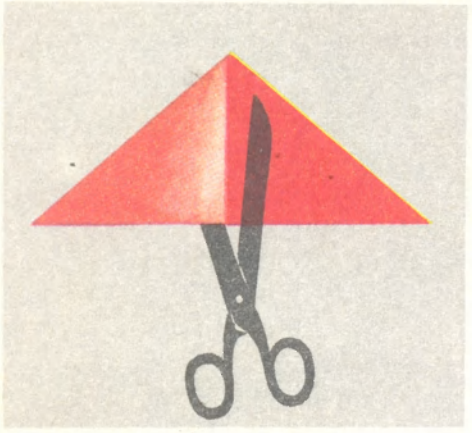


13
Cut an isosceles triangle out of a sheet of paper.

Bend it in two like this:

Now straighten it and cut it along the bend.

6-878



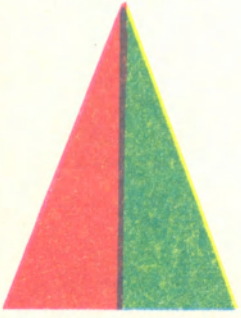
Have a look, you've got two right-angled triangles. Check that they can be put one upon the other so that they coincide. These are equal triangles.

14

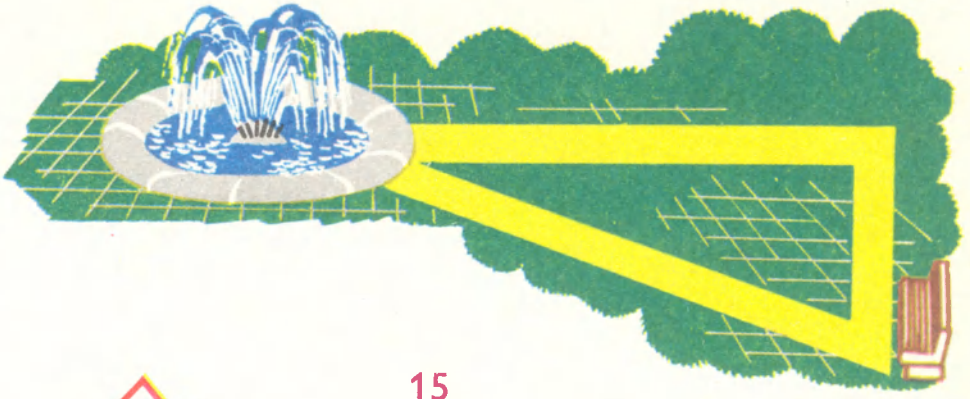
Cut two equal right-angled triangles out of a sheet of paper.

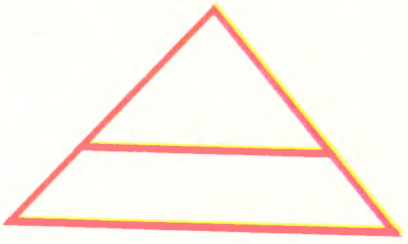
Put them one to the other at first like this:

and then like this:



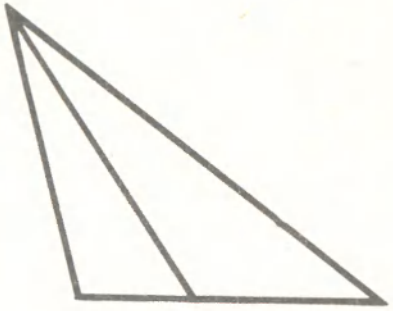
Check that in both cases you've got isosceles triangles.





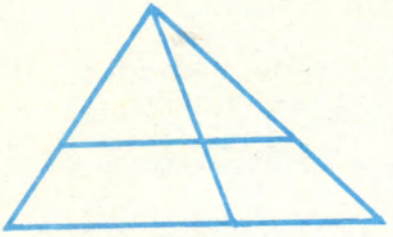
15
There are two paths from the bench to the fountain.
Which is the shorter?

16
There are two triangles in this drawing. Which are they?



There are three triangles here.

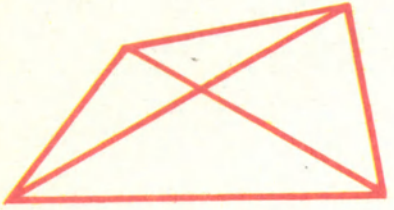
Here they are painted in different colours.



In this drawing there are six triangles:

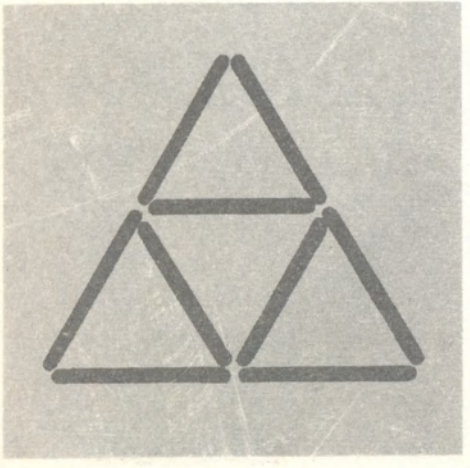
and in this one eight triangles.

Try and find them all.



17

Using three matches you can make up a triangle.
How can you put together two triangles using five matches?



18

Using matches someone has made five triangles.
Point to each of them.
Can you remove three of the matches so that there remains one triangle?

When the Happy gang gathered together again, Dunno told his friends all about his dream. He told them how he travelled across the sea, climbed up the mountains, and met the triangles. He even tried to show how one of the triangles changed its shape. And he remembered the triangle's song.

Dunno recited the song waving his hands.

"But how long are we to remain small? I want to go to school," drawled Pinocchio. 'Let's go to school."

Pencil laughed:

"What are you talking about, Pinocchio. They won't take us yet. We are too small."

"What a pity! Let's go and see what it's like at school."

It was light and quiet in the school. The Happy gang came up to the door of a class-room. Pinocchio opened the door a little and poked his nose warily through the chink.

There were no schoolchildren in the class-room because the classes were over. A teacher sat at her desk and was looking through some exercise-books. She saw Pinocchio and smiled: "Pinocchio, are you here alone?"

"No, I'm with my friends. We came to have a look at school."

"Well, come on in, then. I'm Miss Prism and I know all of you."

Pinocchio, Dunno, Gadgit, and Pencil looked around curiously, and Miss Prism explained:

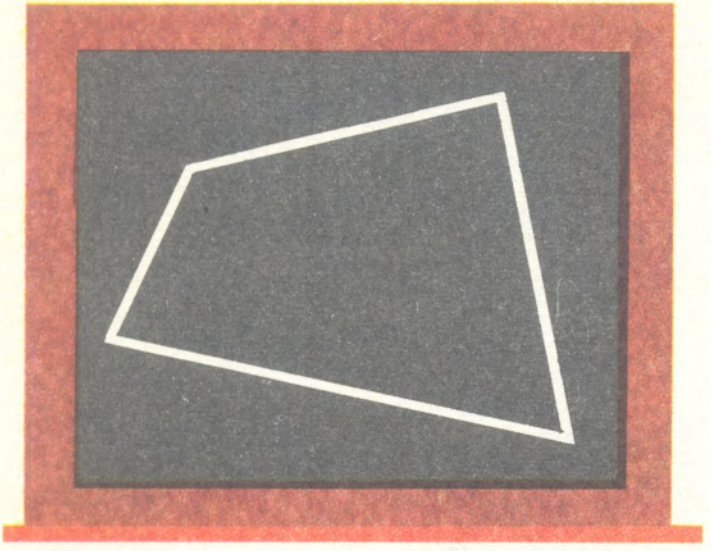
"These are desks. Schoolchildren sit at their desks during the classes. And this is a blackboard and here is some chalk. Teachers often write and draw on the blackboard with chalk. You too may try and draw something with chalk. Go to the blackboard, Gadgit. And let the others sit at the desks."

"As if we are pupils? Shall we play school?" Pinocchio was happy.

"Why not," said Miss Prism, "we might as well play school for a little while. Sit still, and Gadgit will answer at the blackboard. What are you going to draw for us, Gadgit?"

"I'll draw a right-angled triangle."
"Do you really know what a triangle is and what a right angle is?" the teacher was surprised.
"Yes, all of us know. We have been doing some geometry with Pencil."
"I'm glad to hear this. Good boys! Everything you learn will help you when you go to school. What have you learnt already?" Miss Prism turned to Pencil.
"For example, have you said something about quadrilaterals to your friends?"
"Not yet."
"Then I'll tell you about quadrilaterals. We've agreed to play school so I must explain something to you and ask you about it."
"Will you give us marks?" asked Pinocchio.

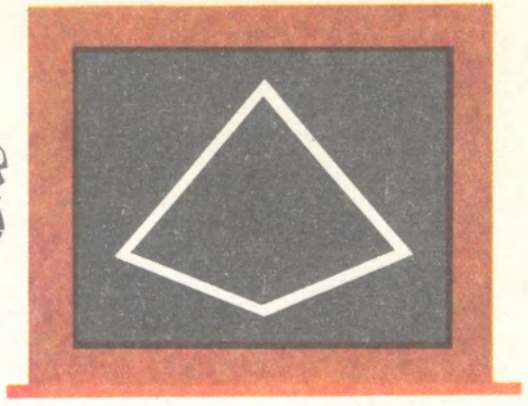
"No, I will not. Let's wait a bit till you really come to school." "Now watch, I'll draw a quadrilateral on the blackboard.
Dunno, why do you think it's called this way?"
"Perhaps, because it has four sides."



Tell, did Dunno answer correctly?

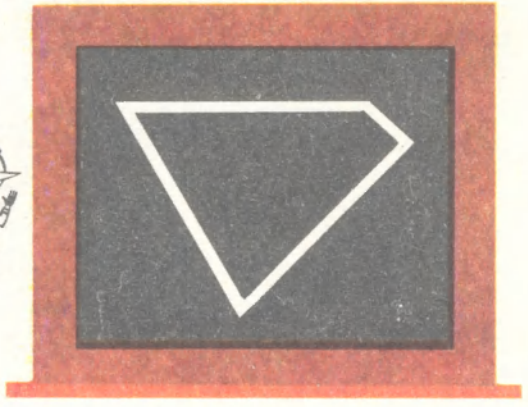
"Right," said Miss Prism, "Gadgit, show us the vertices of this figure. They are called the vertices of the quadrilateral."
"Here they are," said Gadgit pointing. "And these are the angles of the quadrilateral. There are four of them."

Now you point to the vertices and sides of the quadrilateral that the teacher has drawn.

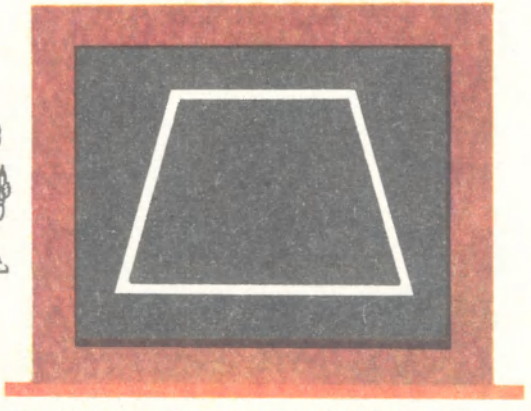


"Good boy," praised Miss Prism.
"Now draw a quadrilateral on your own. Each of you come to the blackboard and draw a quadrilateral on your own."

The first to draw his quadrilateral was Gadgit.

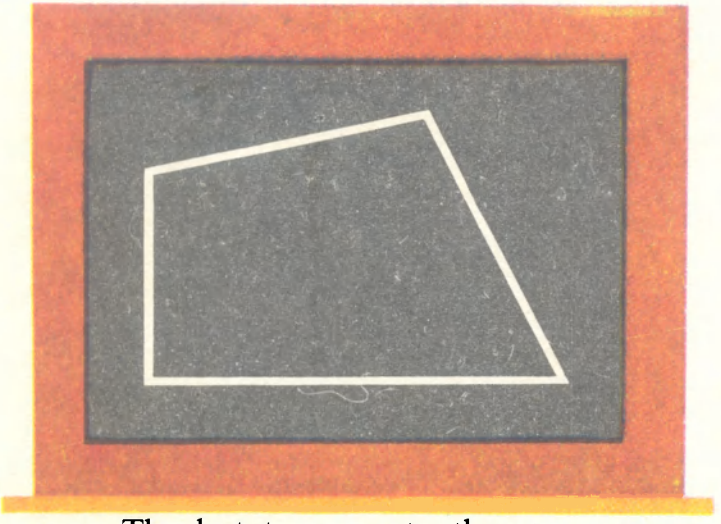


Then came Pinocchio.



Then Pencil.

Now you take a sheet of paper and draw some quadrilaterals.



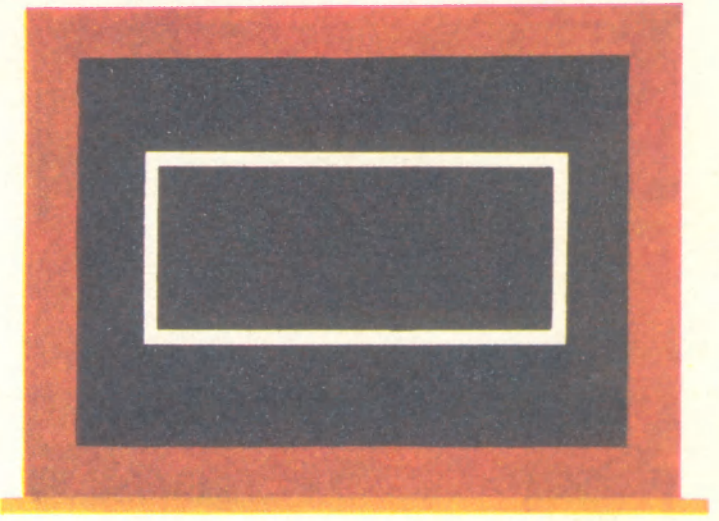
The last to come to the blackboard was Dunno. He walked with a mysterious and important air, and when he got to the blackboard he said: "I have heard somewhere the word 'rectangle'. Now I'll draw a rectangle."

"Why do you think this a rectangle?" Miss Prism was surprised.

"It has a right angle, eh? Here it is."

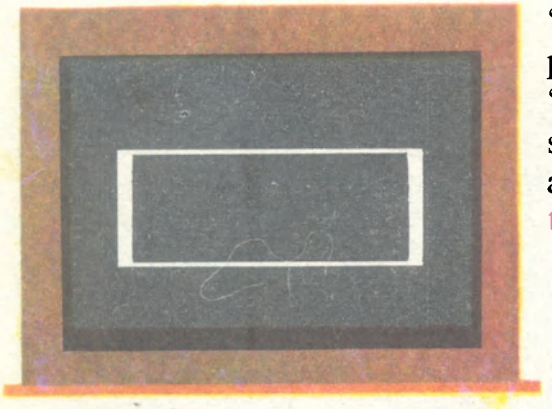
"Yes, only one right angle, but a rectangle (remember!) is a quadrilateral that has all its angles right. Come on, Pencil, draw a rectangle."

Now you try and draw a rectangle (it's best to try this on a sheet of squared paper).

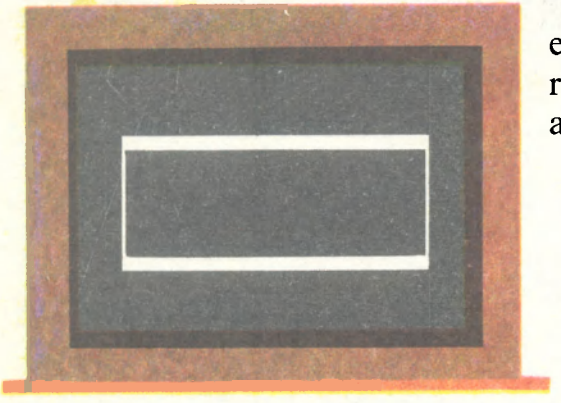


Pencil took a large set-square and drew:
"This is a rectangle," he said. "All its angles are right."
"All right," said Miss Prism, "now look around you and name some objects that are rectangles."
"A window, the door, the blackboard," Pinocchio, Dunno and Gadgit began to enumerate interrupting each other.

Now look around you and name some objects that are rectangles.

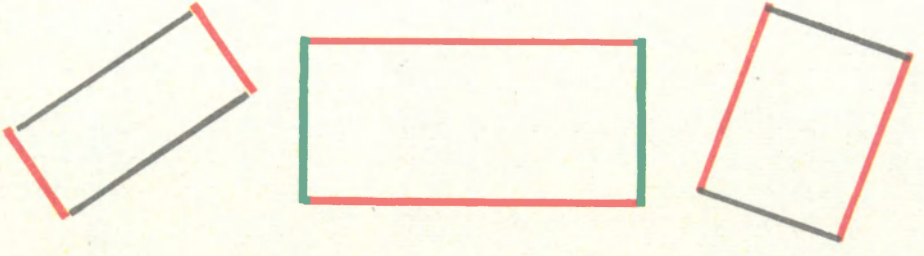


"Notice," said Miss Prism, pointing at Pencil's drawing, "in a rectangle these two sides have the same length, or as is usually said, are equal to each other."



"And these two sides are equal too. In general, in any rectangle the opposite sides are equal."

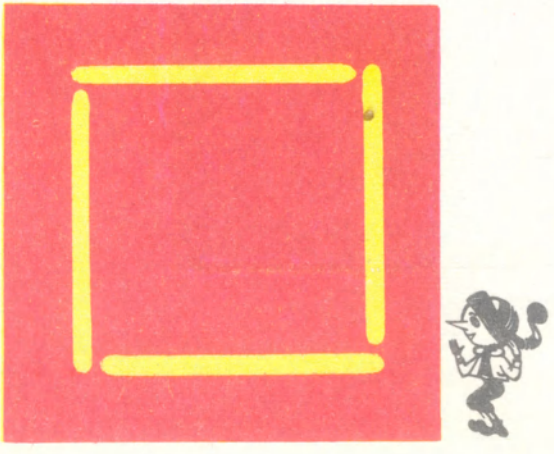
Here are several rectangles. Their opposite sides are of the same colour. Check that the opposite sides are equal.



"And now we'll make some rectangles out of sticks. What sort of sticks will we need?" asked Miss Prism. "If you know raise your hand."
The first to raise his hand was Gadgit.
"Two must be one length and another two must be another length," he said.
"Correct," said Miss Prism. "Here are such sticks. Make a rectangle out of them."

Now you make up a rectangle out of sticks (don't forget that each corner must be a right angle).

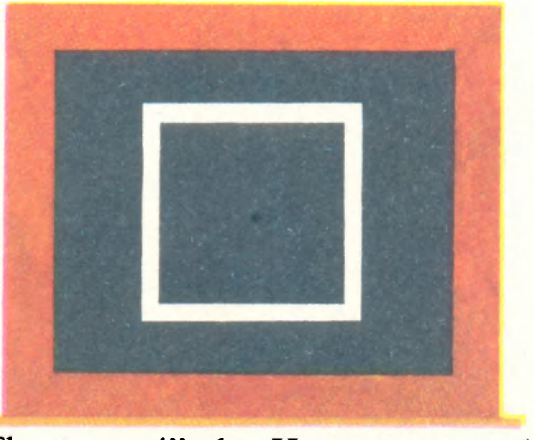
Suddenly Pinocchio began to fidgit and first raised one hand and then put up both his hands.
"Miss Prism, Miss Prism! I also want four sticks, only all the same length please. I'll make a rectangle out of them. I will get a rectangle, won't I?"



Of course you will, only with four equal sticks you will get more than just a rectangle."
"I've done it already!" cried out the happy Pinocchio. "Look, a rectangle with all of its sides equal. It's obviously called an equilateral rectangle."

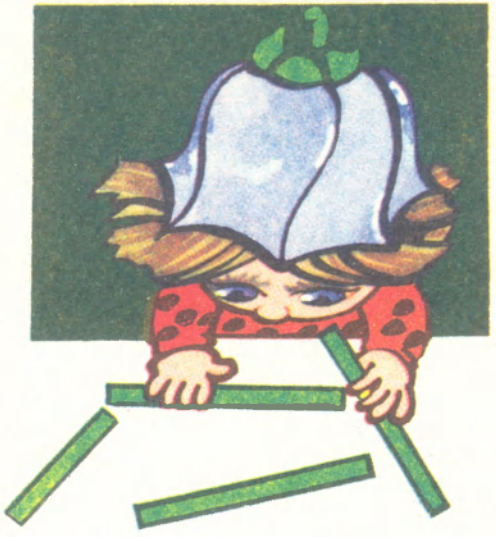
"It's not normally called that," said Miss Prism. "There is a special name for a rectangle with all its sides equal. It's called a square. So you've constructed a square."
Now Pencil raised his hand.
"Miss Prism, I know a geometrical puzzle. May I tell you it, please?"
"Of course, please do. We shall all listen to your puzzle with pleasure."
Pencil stepped out of his desk and began to recite loudly:

Know of him you might,
Each angle of him is right,
Each side of him is like the others
Four identical brothers,
Perhaps, you have met him
somewhere,
This is my friend ...



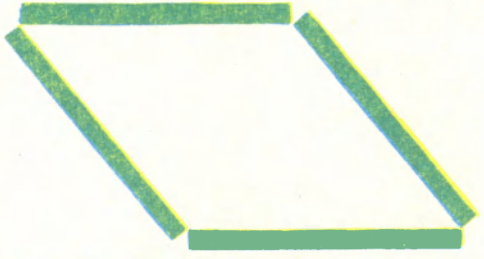
"The square!" the Happy gang ended in chorus, and Pencil went to the black board, took the chalk and drew a large square.

Now you draw some squares on a sheet of squared paper. Paint them with different colours.



"I want four equal sticks too," said Dunno. "I want to make a square."
Miss Prism said:
"I'll give you the sticks, Dunno. But we've decided to play school, and so you must remember that when you want to say or ask something in class you must raise your hand first."
"Agreed," said Dunno and raised his hand immediately.
"Well," smiled Miss Prism, "What do you want to say?"
"I want to make a square out of sticks."

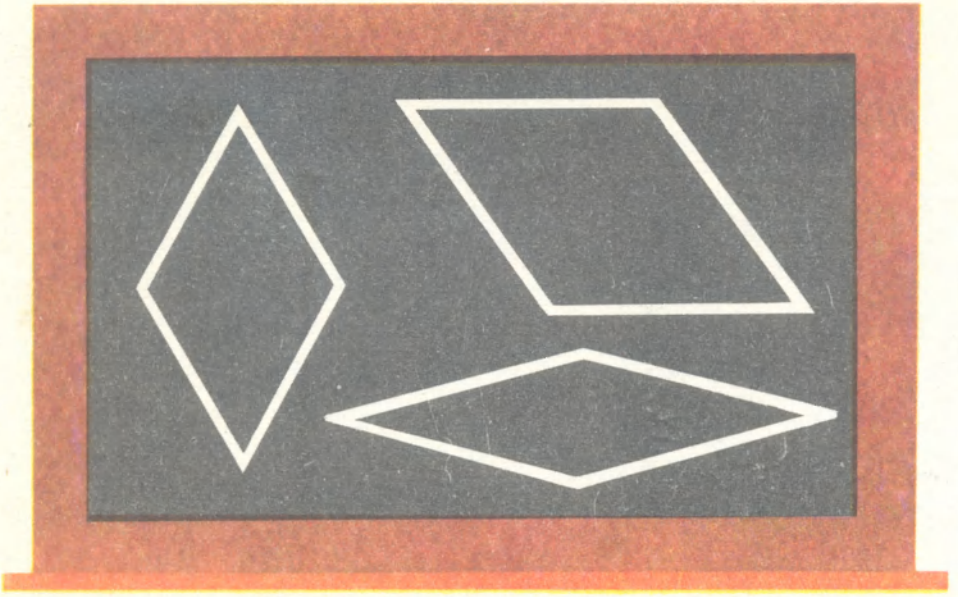
Now you take four equal sticks and make up a square.



Dunno didn't get a square. Here is the figure he made:

Can you think why this figure cannot be called a square?

"Dunno, you haven't got a square," said the teacher.
"Why? All the sides are equal."



"What of it. Not every quadrilateral with equal sides is called a square. You forgot about the corners. They must be right angles. In your quadrilateral the corners are not right angles. Therefore it cannot be called a square."

"But what is it called?" asked Dunno and looked cautiously at Pinocchio. "You, Pinocchio, will again tease me that I'm like the little Point in the fairy tale?"

"Okay, I won't," promised Pinocchio and Miss Prism got interested at once:

"Which fairy tale are you talking about, Dunno?"

"The geometrical one. Pencil told us about a Point who was travelling in Geometry Land. Together with Compasses it went to the Town of Triangles. When the triangles learned that Compasses and Point were looking for Rubber-Robber they asked to take them as assistants. So they decided to go on together to catch and punish Rubber-Robber."

"It's very interesting," said Miss Prism. "I'd like to listen to this fairy tale about Point."

Gadgit quickly raised his hand.

"What do you want to say, Gadgit?"

"May Pencil go on with the fairy tale now? It's a long time already since we heard the last bit."

"I agree. Let's ask Pencil to go on with his fairy tale. But first I have to answer Dunno's question. Remember he asked me what the quadrilateral with equal sides is called. It is called a rhomb or rhombus. I'll draw some rhombs on the blackboard."

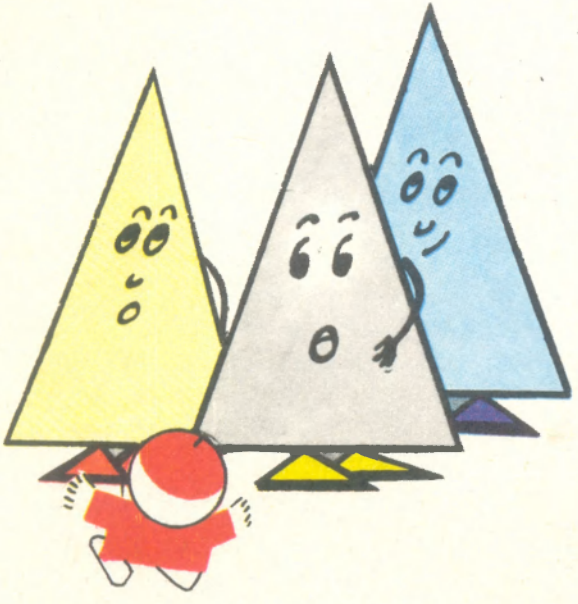
Now you make a rhomb out of sticks.

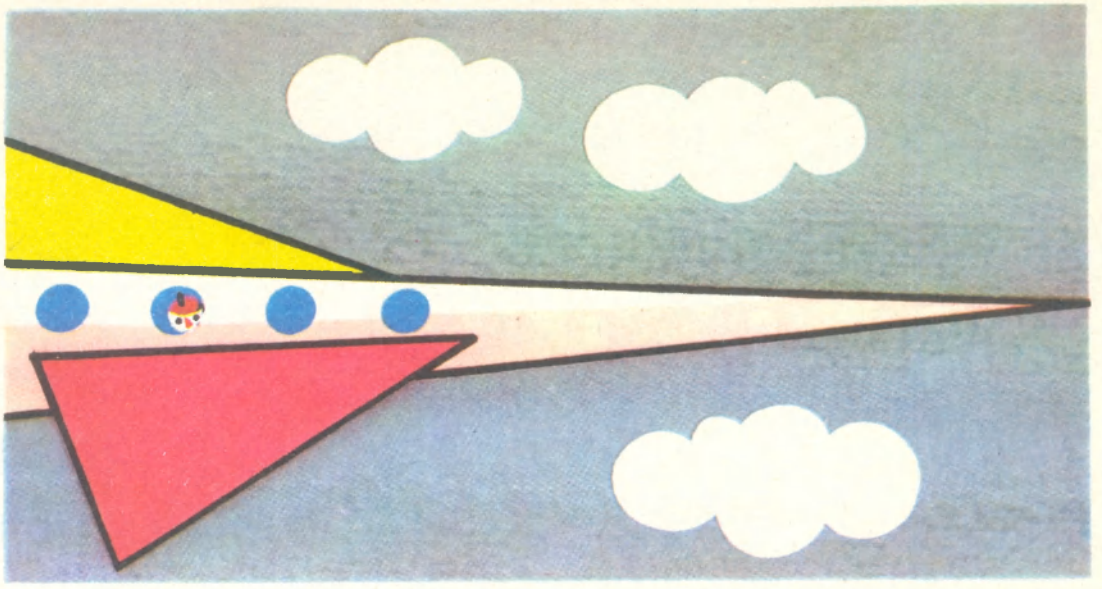
Miss Prism put down the chalk.
"Well," she said, "we have had a sort of a lesson with you. At the end of a lesson the teacher always gives the schoolchildren some homework. I will also give you some homework. What can you say about a rhomb with right angles?"

You too think about this question.

"Now to the fairy tale. We're listening to you, Pencil."
Everybody made himself comfortable and Pencil began to talk.

The triangle-builders said:
"We must teach the villain a lesson. Let's join forces. Will you take us as assistants?"
"Of course," said Compasses, "let's all walk together."
"No," said the triangles, "walking would take too long. After all, we could travel much faster."
"But how?" cried Compasses and Point in unison.
"By flying in an aeroplane."
"Hurray!" Point was happy. "I've never flown in an aeroplane before, Is it dangerous?"
"No," Compasses comforted Point. "On the contrary, it's very interesting. Let's hurry up and go to the aerodrome."
The plane was already about to start and with its triangular wings pressed to its sides it seemed to want to rush forward. Point, Compasses and their new friends got into the plane, the plane roared, ran along the runway, jumped into the air and began to gain height quickly.
Point looked curiously through a porthole. Down below the





straight lines of roads and meandering lines of rivers and streams ran in different directions. Here and there between the lines squares of houses and rectangles of gardens were to be seen.

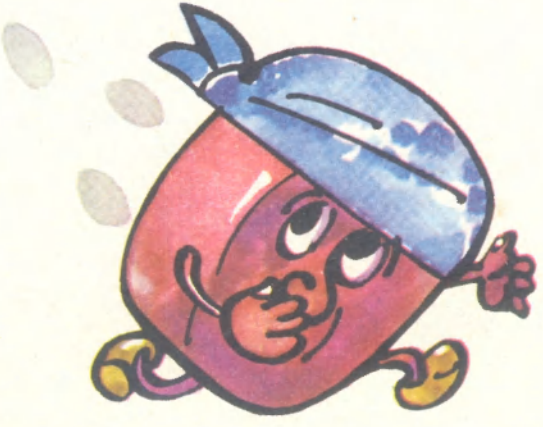
Suddenly Point noticed that one of the squares had disappeared somewhere. Then another.

"Look," she cried, "the squares over there are vanishing."

"We'd better fly a bit lower and see what is happening," suggested Compasses.

The plane came down and everybody saw Rubber-Robber. He was again doing mischief, mercilessly erasing houses.

"There he is! Let's get him!" cried out Point.



Everybody jumped and cried: "We've got you! You won't escape!"

The pilot directed the plane at Rubber-Robber. Rubber-Robber saw his pursuers and took to his heels. He was running fast but the plane was flying even faster. It was

about to catch up with the villain, but suddenly the plane touched a tree with its wing, rolled and lost speed. Rubber-Robber got away.

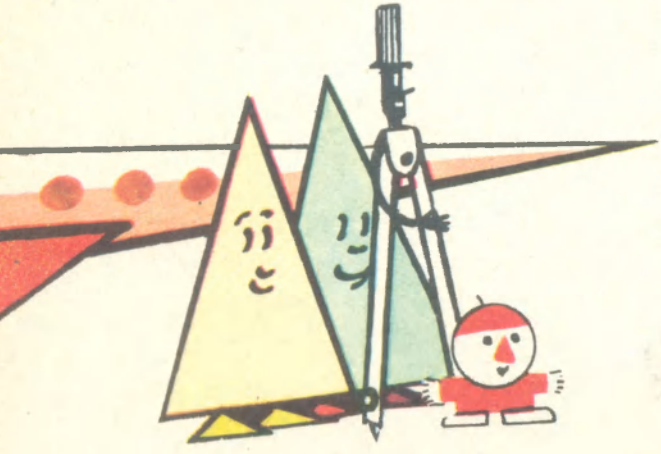
"What's the matter?" everybody was worried.

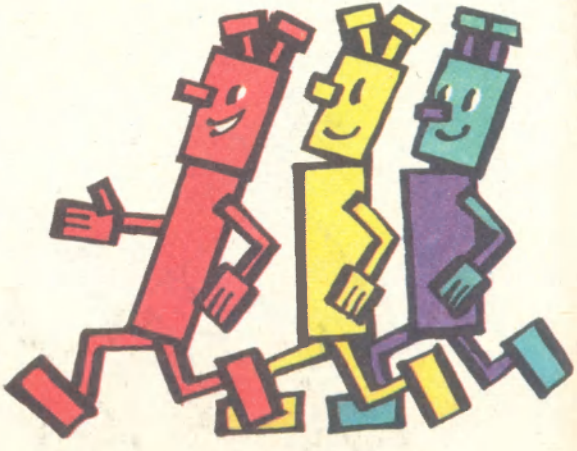
"The wing of our plane is damaged," explained the pilot. "We'll have to land quickly. Is there an aerodrome nearby?"

"I can see a town over there," pointed Compasses. "It's bound to have one."

"We'll fly there," said the pilot. The plane only just made it to the aerodrome and landed. All the people who lived in the town came to see the travellers. Point and her friends immediately noticed that everybody here was made of quadrilaterals.

"We're glad to welcome you to our Town of Quadrilaterals and we hope we can help you," said the quadri-





lateral people.
"So we've come to the Town of Quadrilaterals," exclaimed Point. "It's so interesting! I didn't know that there was such a town. We've come from the Town of Triangles. We're out to catch Rubber-Robber."
"Catch Rubber-Robber?" said the quadrilaterals. "We've heard of him and his mischief. He must be caught by all means. Could we help you somehow?"

"We need to have a new wing on our plane," said the pilot. "Could this be done in your town?"

"Why, of course. Come to the factory where our aeroplanes are made. There are many different sorts of wings there."

So all of them wanted to visit the factory.

On the way Point was looking around curiously.

"Just look. Compasses," she said with surprise. "In this street almost all quadrilaterals are similar: their angles are right."

"No wonder," said Compasses. "The street we are walking down is called the Street of Rectangles."

"Have you got a Street of Rhombs in your town?" Point asked the quadrilaterals.

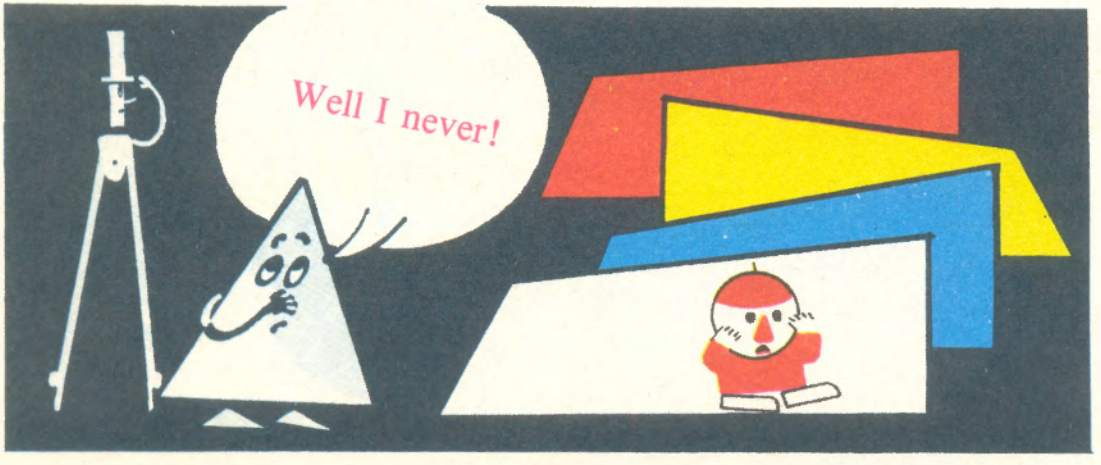
"Yes, we have. It's not far from here," answered her new acquaintances.

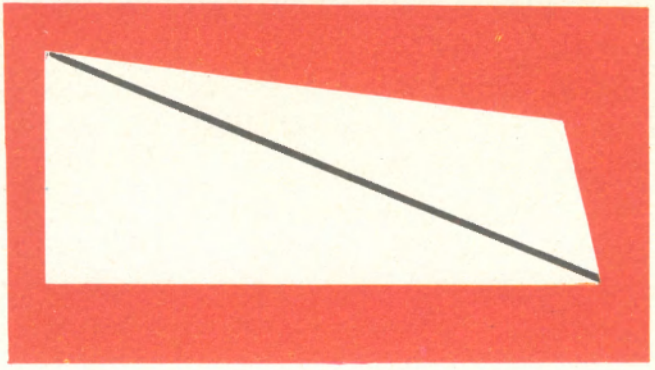
"Perhaps, you've got a Street of Squares?"

"No, there is no special street of squares. Squares can live both in the Street of Rectangles and in the Street of Rhombs."

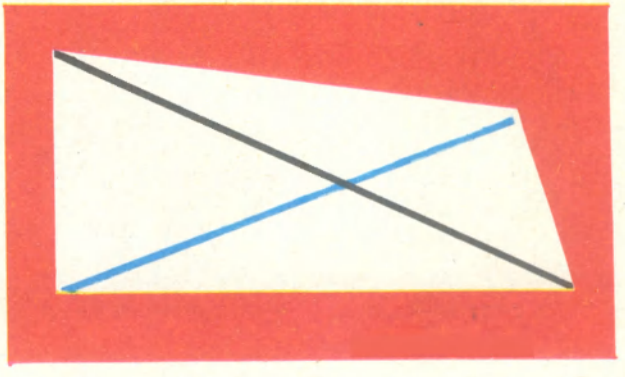
"But why...," Point began to ask, but Compasses interrupted her. "I'll explain it to you later. We shouldn't waste time, lest Rubber-Robber gets too far away. We have to hurry to the factory."

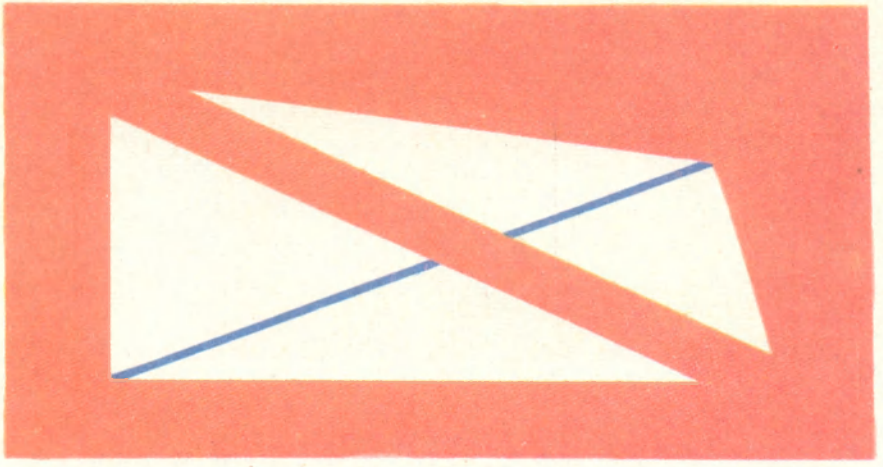
It turned out that there were many various wings at the factory but... all of them were quadrilateral.





"Well I never," said the pilot in bewilderment. "These wings won't do. Our plane is from the Town of Triangles and it won't fly with a quadrilateral wing. It needs triangular wings."
What was to be done?
Nobody could think of anything. Then Compasses suggested: "Let's call in the scissors. He'll invent something."
The scissors came, found out what the problem was and said:
"It's child's play. A quadrilateral wing must be cut along a diagonal to give two triangular wings."
"I don't understand. What do you mean to cut along a diagonal?" asked Point. "What's a diagonal?"
"Just a minute," said the scissors. "Look, here is a wing of a plane. It's quadrilateral in shape. I'll call in a segment to connect opposite vertices of the quadrilateral... Ready! This is a diagonal of the quadrilateral."
"I see," said Point. "A diagonal connects opposite vertices."



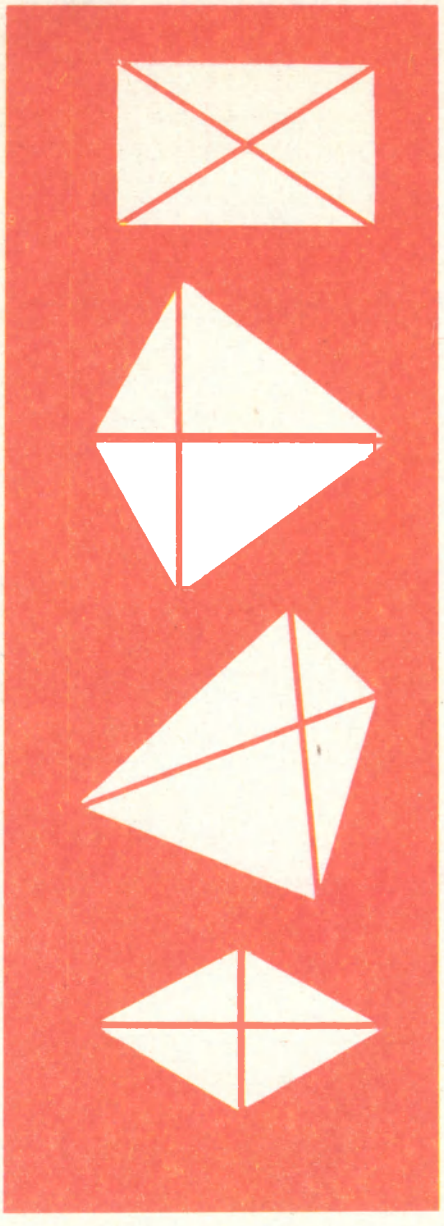


"Have a look," continued the scissors. "There is another pair of opposite vertices in a quadrilateral. They can also be connected by a diagonal."

"A quadrilateral has thus got two diagonals?" asked Point.

"Yes," said the scissors, "and now we'll cut the quadrilateral wing along one of the diagonals. Here you've got two triangular wings. Take any one."

The quadrilaterals quickly replaced damaged wing by a new one and the plane was ready to carry on with the flight. The travellers thanked their kind hosts from the Town of Quadrilaterals and the scissors for help. Then Point, the Compasses, and the triangles got into their plane. The scissors too decided to take part in the pursuit and went on board the plane. The plane took off



and flew on in search of Rubber-Robber.

At this point Pencil interrupted his story.

"You know what, friends," he said, "it's high time we were going. We've taken enough of Miss Prism's time as it is."

"Nothing of the sort," said Miss Prism, "I enjoyed talking to you. And your tale is interesting."

And the tale told us
Where live diagonals

sang Pinocchio and looked cunningly at the teacher. Miss Prism laughed.

"That's very good, Pinocchio. So where do they live, those diagonals?"

"In a quadrilateral. There are two of them, and they connect the opposite vertices."

"Right you are indeed. Here are some quadrilaterals with their diagonals: Gadgit asked:

"Miss Prism, are there any five-sided figures?"

"Yes, they are called pentagons."

"And six-sided?"

"Of course, they're called hexagons."

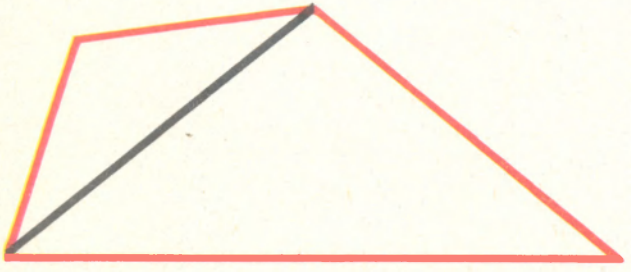
"But how many diagonals have pentagons and hexagons got?"

"You see, Gadgit, it's a rather difficult problem. When you become a school-boy, you'll know more and then you'll be able to solve it. And you'll be able to do many, many other more difficult problems."

WORKSHEET

1

Draw a quadrilateral. Point out its vertices and sides. Draw in its diagonals.



2

Cut a quadrilateral out of a sheet of paper. If you now cut it along the diagonal, you'll have two triangles. By cutting a rectangle along a diagonal you'll obtain two right-angled triangles. What triangles will you get if you cut a rhomb along a diagonal? Or a square? (Answers: isosceles ones; right-angled isosceles ones.)

3

Show that by cutting a rectangle or a rhomb along a diagonal you'll always get equivalent triangles. You can easily check this by putting one triangle obtained upon the other.

4

Cut two equal right-angled triangles out of a sheet of paper. Put them together so that the result is a rectangle.

5

Cut two equal isosceles triangles out of a sheet of paper. Put them together so that the result is a rhomb. What sort of triangles are needed to obtain a square?

6

You can say that any square is a rectangle, but can you say that any rectangle is a square?

7

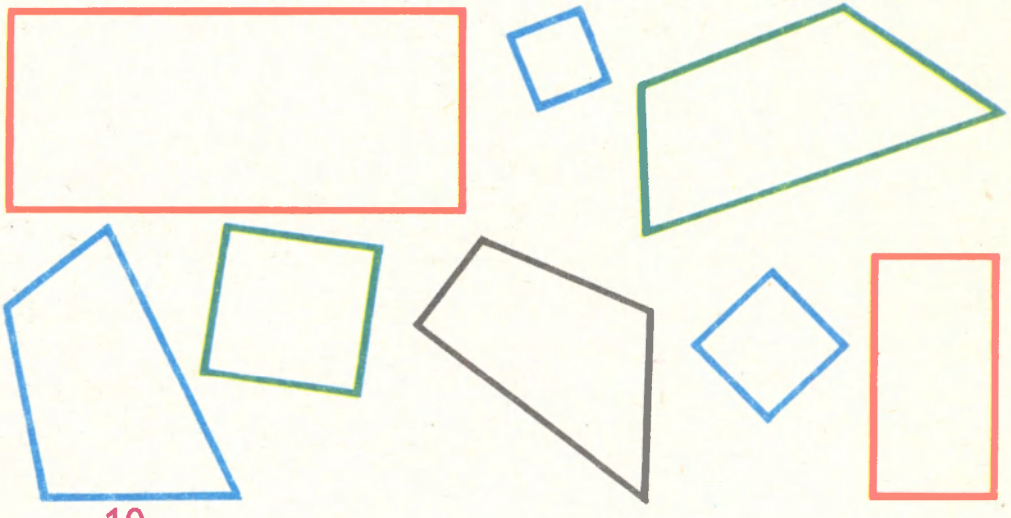
You can say that any square is a rhomb, but can you say that any rhomb is a square?

8

Draw a rectangle that is not a square. Draw a rhomb that is not a square.

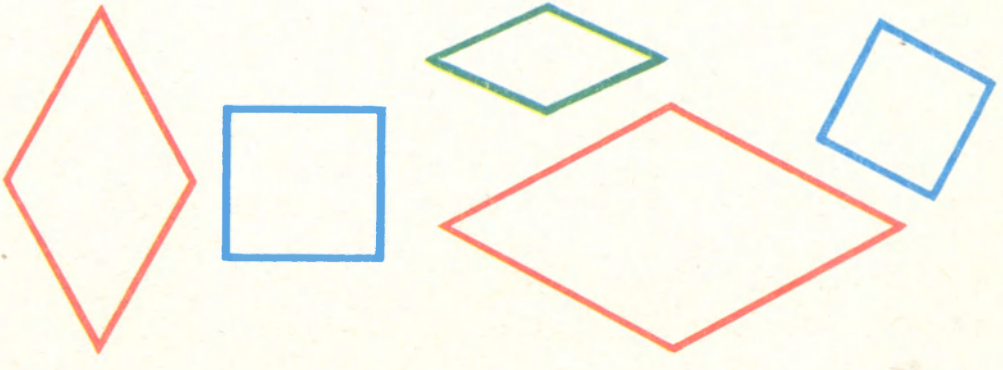
9

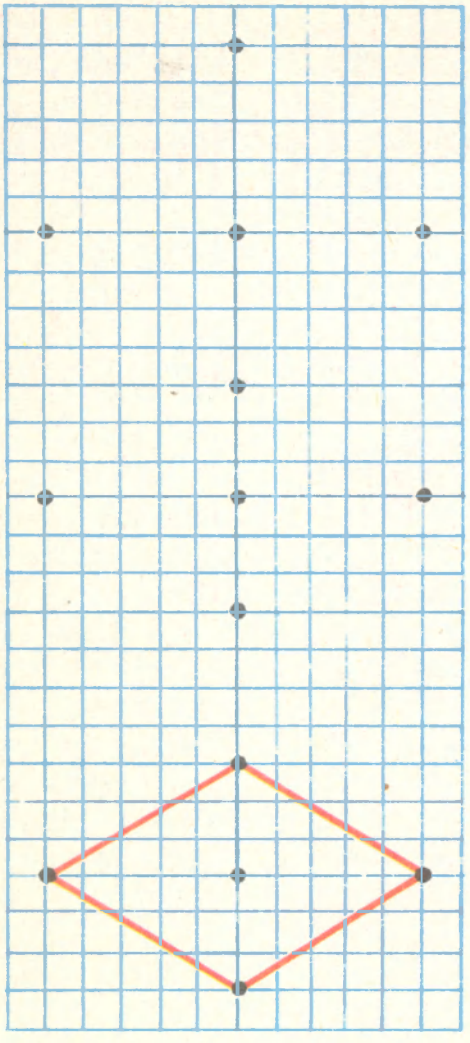
Here are some quadrilaterals:
Count the rectangles. Count the squares.



10

Here are some rhombs:
Count them.
How many squares are there?





11

Put a point on a sheet of squared paper.

Count the same number of squares on either side of the point and put points. For example, like this:

Now count the same number of squares upwards and downwards from the first point and again put points. For example, like this:

Now connect the four points obtained like this:

Check if the quadrilateral obtained is a rhomb.

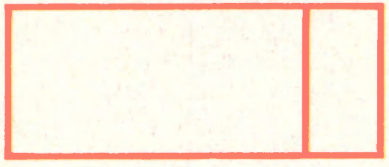
Rhombs can easily be obtained on a sheet of squared paper. Draw several rhombs.

12

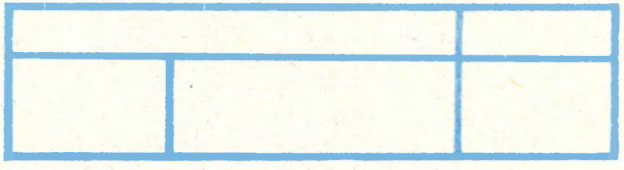
On a sheet of squared paper put a point and then draw a rhomb as before. Draw in the diagonals. Notice that they intersect at the point where you began your drawing from. What angles are formed at the intersection of the rhomb's diagonals?

13
Make up a quadrilateral out of sticks. Can you do this out of any four sticks?

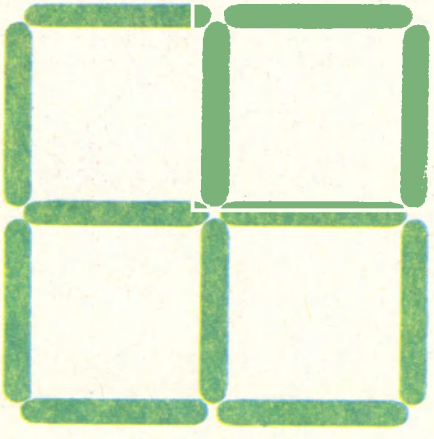
14
Choose four sticks such that no quadrilateral can be made of them.



15
There are three rectangles in this drawing. Point them out



16
How many rectangles are there in this drawing? (Answer: seven.)



17
Use counting sticks to make up this figure:

There are five squares here. Point to them. Which two sticks have to be removed to leave three squares? Two squares?

The Happy gang have remembered their visit to the school for a long time. Now in their geometry "classes" they behaved as if at a lesson. If, for example, one of the friends wanted to ask or say something, he raised his hand and waited for Pencil to ask him.

Once Gadgit raised his hand and asked:

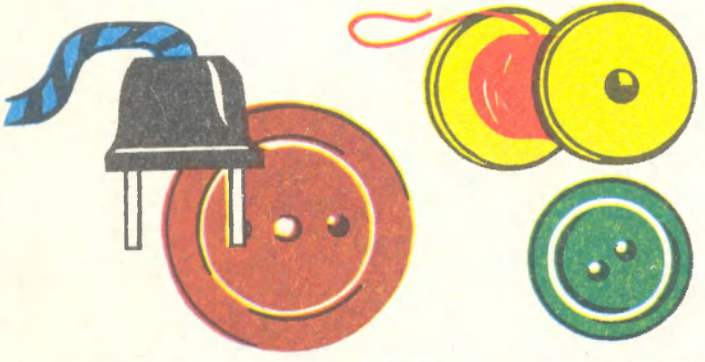
"Pencil, will you tell us about circles?"

"I was just going to speak about circles today," said Pencil. "The circle is an important figure in geometry. Many objects are circular in shape. Can you name some of them?"

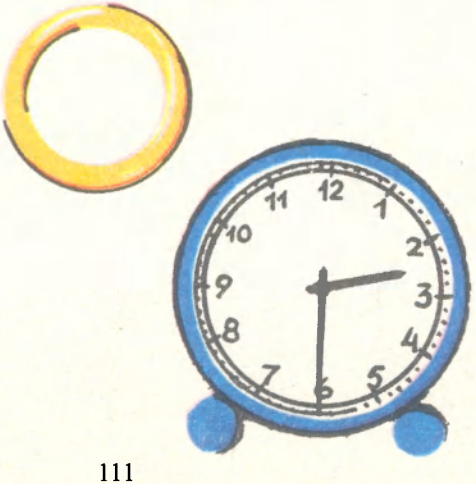
Gadgit said a saucer.

Pinocchio said coins and a drum.

Dunno said a hoop and a clock.

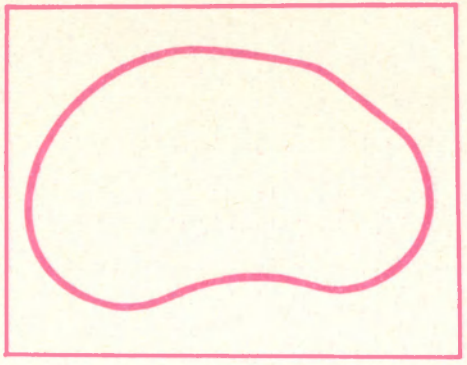


Now you name some objects that are circular in shape.



Pencil was pleased:
"Yes, those are all circles. We could think of many examples: the cover of a saucepan, a plug, a button... But circles are especially widely used in engineering. I have even invited an engineer to come today, and asked him to tell us about circles in much detail.

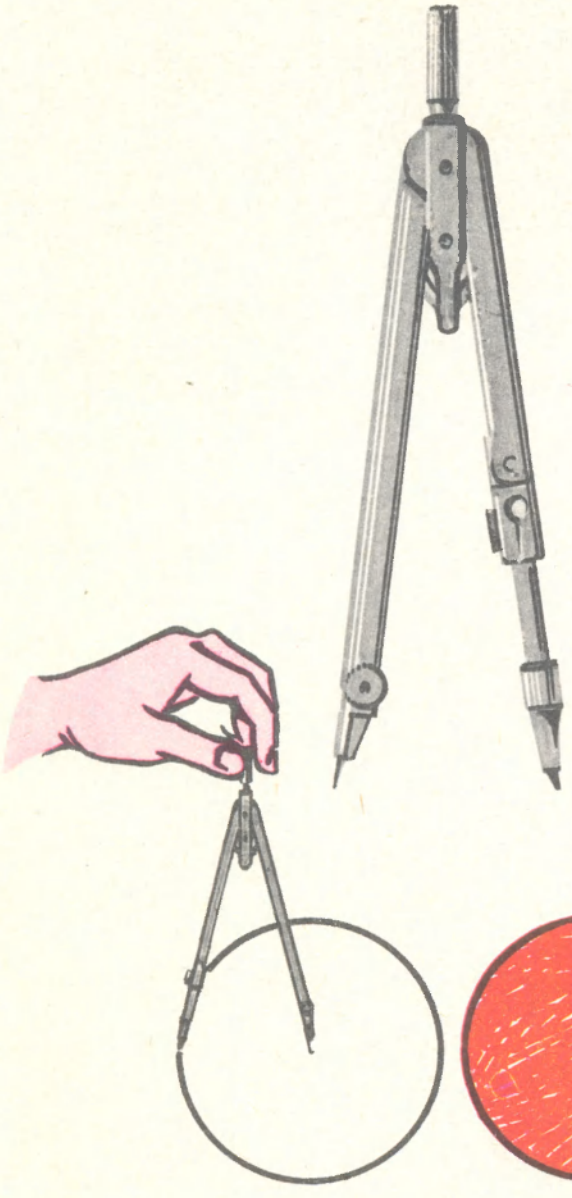
No sooner had Pencil said this that the door opened and the friends saw a tall smiling man. He had a large bag in his hands.
"Hello, friends. I am the design engineer."
"Hello," answered the Happy gang, and Pinocchio asked. "What's your name?"
"You can call me simply Uncle John."
"Will you tell us about circles, Uncle John?" Pinocchio went on to ask.



"I'll do better and show you as well," answered the engineer. "Which of you can draw a circle on a sheet of paper?" "I can," boasted Dunno.
And so he did.
Uncle John smiled:
"You've got not a circle but some potato. Nothing doing. How about the others, what about you?" he looked at the rest of the Happy gang.

? "What can you suggest?"

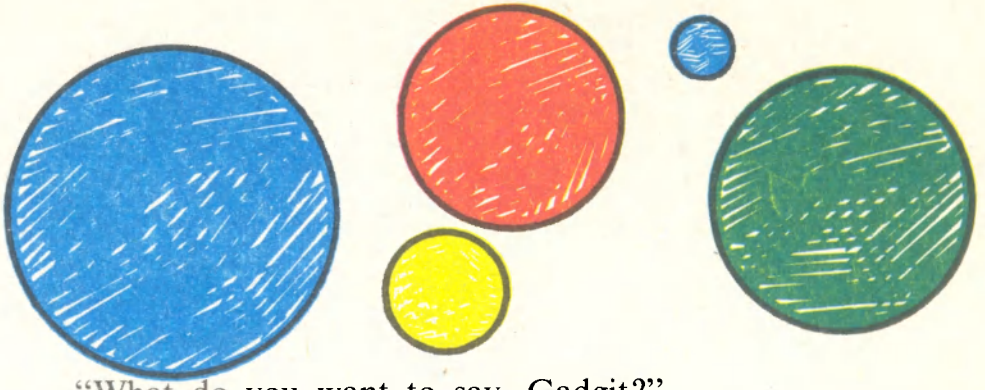
Gadgit said:
"A circle can be drawn by putting a saucer or a plate on a sheet of paper and tracing round its edge with a pencil."
"Not bad. But that's not very convenient. Suppose you have to draw many different circles, large ones and small ones. Surely, you wouldn't have a heap of saucers about you."
"In order to draw a circle you should...," slowly began Uncle John looking at Pencil as if suggesting to him



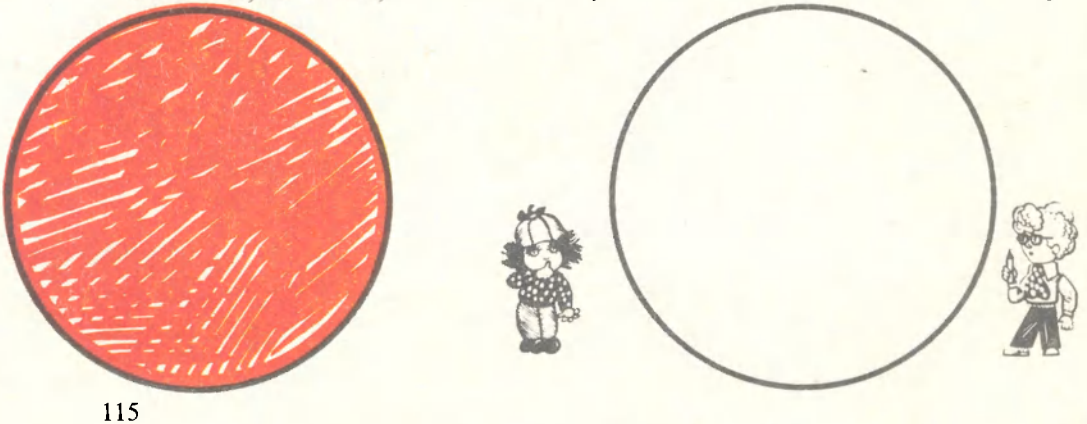
to continue, "...you should use a pair of compasses," ended Pencil.
Uncle John nodded in consent, unbuckled his big bag, and produced a pair of compasses. He showed them to the Happy gang. "Are these dividers?" asked Pinocchio.
"Not exactly," said the engineer. "In this pair of compasses there is a needle only at one end, and at the other a pencil. Look, I put the leg with the needle on the paper and draw a circle using the leg with the pencil."
"Now I'll paint it."
"The legs can be pulled wider apart to make the circle larger. If we push the legs together, the circle will be smaller."

Now you take a pair of compasses with a pencil and draw some circles using it. Paint them all if you want.

Suddenly Gadgit raised his hand.
"Oh, I see you behave properly here," said Uncle John.



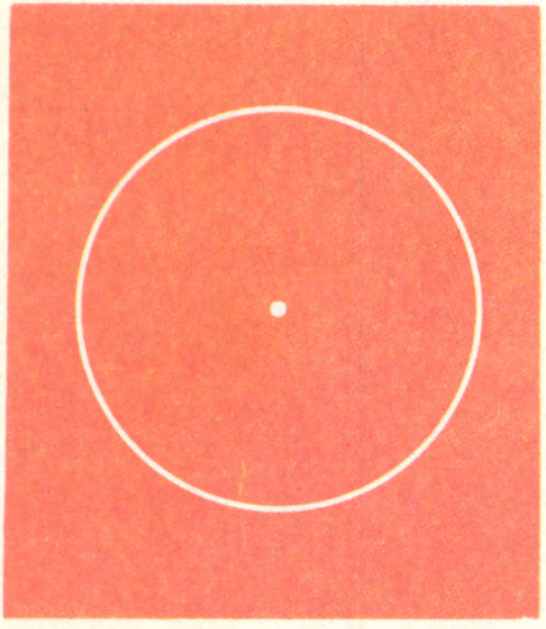
"What do you want to say, Gadgit?"
"What is the name of the line that a compasses draws?"
Dunno even opened his mouth in surprise: the clever Gadgit asking such a silly question!
"Well, it's a circle. It's called a circle," Dunno began to explain to Gadgit. "No need to ask, it's very simple."
"Stop, Dunno," interrupted the engineer, "you are wrong. Gadgit did ask a good question. It isn't as simple as that. Look:
a circle here is all that is painted.
But the line that a compasses draws is called a circumference."
"Do you see now, Dunno? Although, sometimes circumference is called a circle, too."
"You see, Dunno," said Pencil, "a circumference runs along



the edge of a circle. Isn't that so, Uncle John?"
"Right. In geometry books, they say that a circumference is the line that bounds a circle or a ring. Now friends, take my compasses and draw some circumferences."

! Now you draw some circumferences with a pair of compasses.

Gadgit again raised his hand. Uncle John looked at him approvingly:
"What do you want to ask now, Gadgit?"
"Uncle John, when we draw a circumference with compasses, the needle always leaves a point on the paper. What is it called?"



"The centre of the circle, though some people call it the centre of the circumference. Come on, Pinocchio, show us the centres of all the circles and circumferences that we have drawn. You have been silent for too long and looking sideways."
"Perhaps he is composing a song about circumferences," said Pencil. "He is always making up songs about the new things."
"So that's it!" said Uncle John. "Well, what have you thought of now?"
Pinocchio was taken aback. He had been thinking about something completely different and didn't know what to say.

"I... I haven't thought of anything yet," muttered Pinocchio, "but I can... if you like, he went on to say having gained a bit of courage.

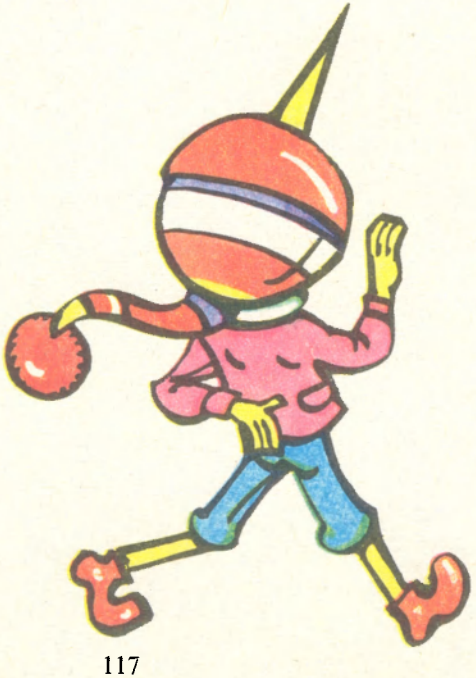
"You'd better invent a puzzle about circumferences, not a song," said Dunno.

"But why should I make up riddles about it when we all know the answer? I'd rather invent the answer to a riddle about it."

Pinocchio stood up and began to walk to and fro. He closed his eyes, looked at the ceiling, muttered something, waving his hands and rolling from side to side.

"Ready!" he cried out at last.

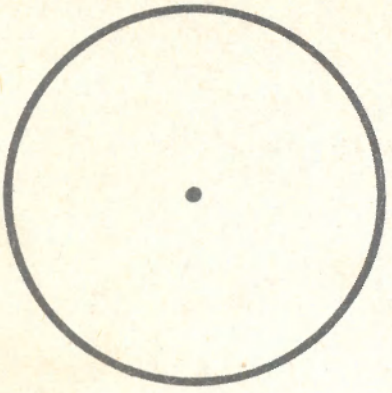
The ring has a close friend,
With him ring's always in conference.
He runs along the ring's rand
And is called circumference.



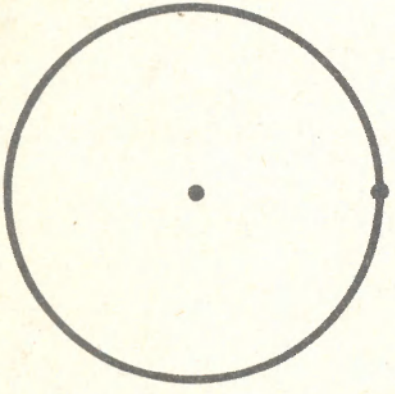
"Very good! What a smart boy you are," Uncle John was surprised. "How did it go? ... 'With him ring's always in conference... and is called circumference'. What else do you know about circumferences?"

Pinocchio, Gadgit and Dunno were silent.

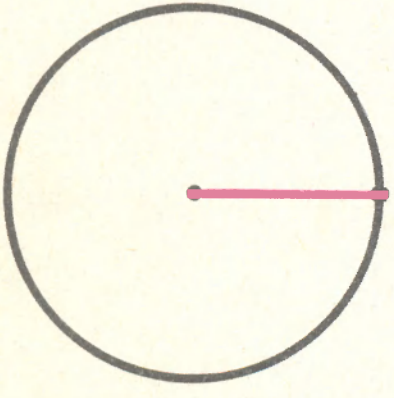
"Well then," said the engineer, "Pencil, you'll have to help your friends. Come on, tell them what a radius is?"



Pencil drew a circle, marked its centre,



then marked a point on the circle



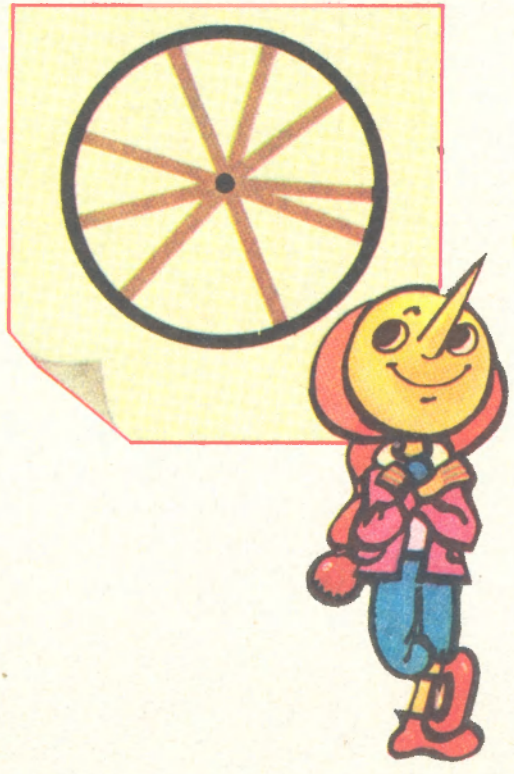
and connected the point with the centre. Like this:

"This segment is the radius of the circle," said Pencil.

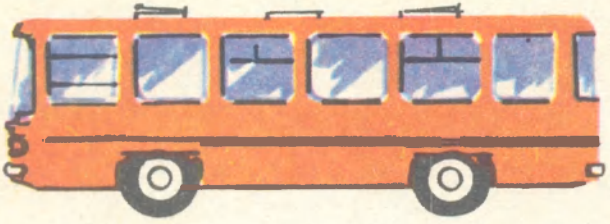
Now you do the same drawing and draw in a radius of the circle.

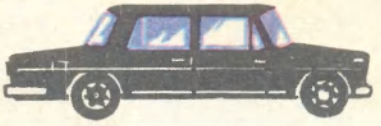
"Well," Uncle John was pleased, "you see, boys, a radius is a segment that connects any point on the circumference with its centre."
"So you can draw very many radii," said Gadgit.
"Of course. Take any point on a circumference, connect it with the centre and you'll get a radius. Come on, try your hand at it. Draw some circles and their radii. Notice that in a circle all its radii are equal."

Now you draw a circle and draw in several radii. Check if they are all the same length.

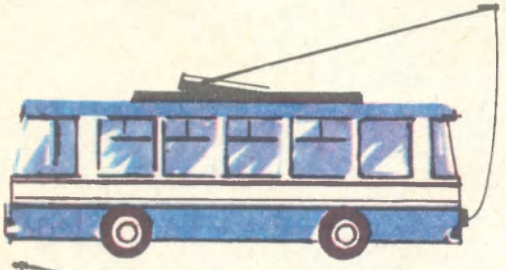


Now Pinocchio raised a sheet of paper on which he had drawn his radii.
"Look what I've got?" he cried out, "just like a wheel with spodes."
Uncle John looked attentively at Pinocchio and said:
"A wheel is a very good idea. It has the shape of a circle and in engineering you cannot do anything without wheels. So the wheel is very important in engineering. If something spins or rolls you are sure to find a circle.

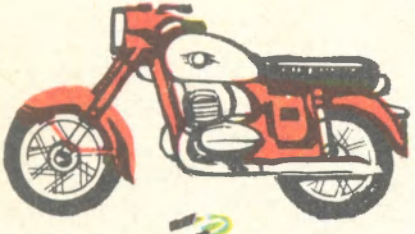




Cars roll on wheels.

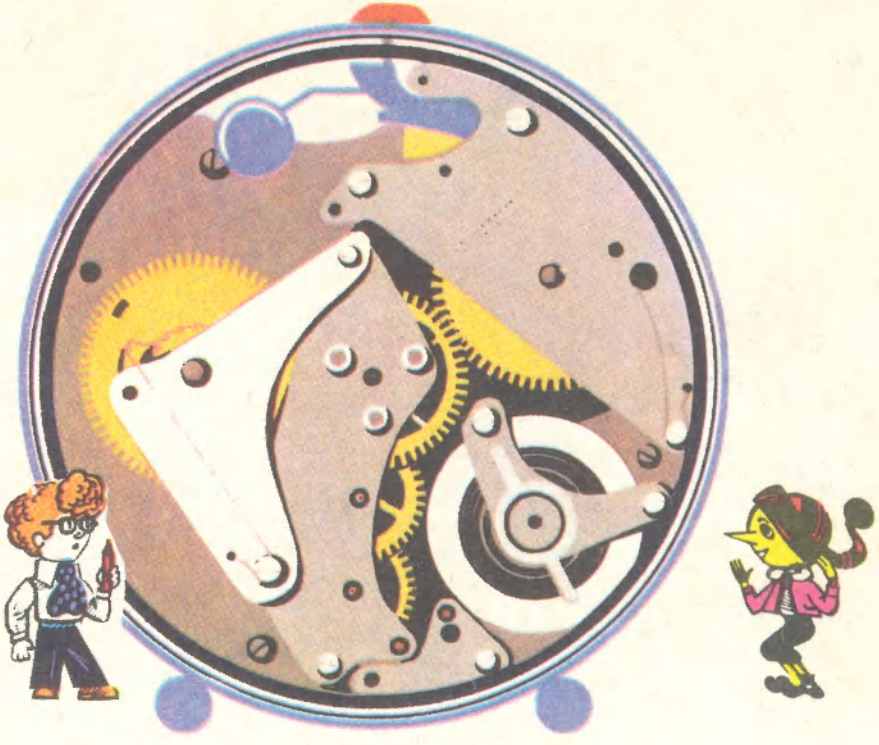


Trams and trolley-buses roll on wheels.

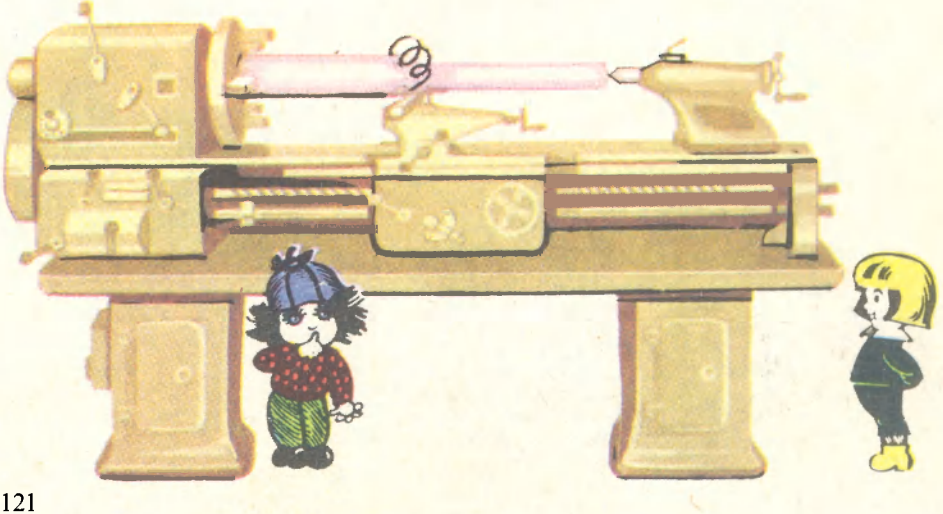


Motor-cycles and cycles are also on wheels.

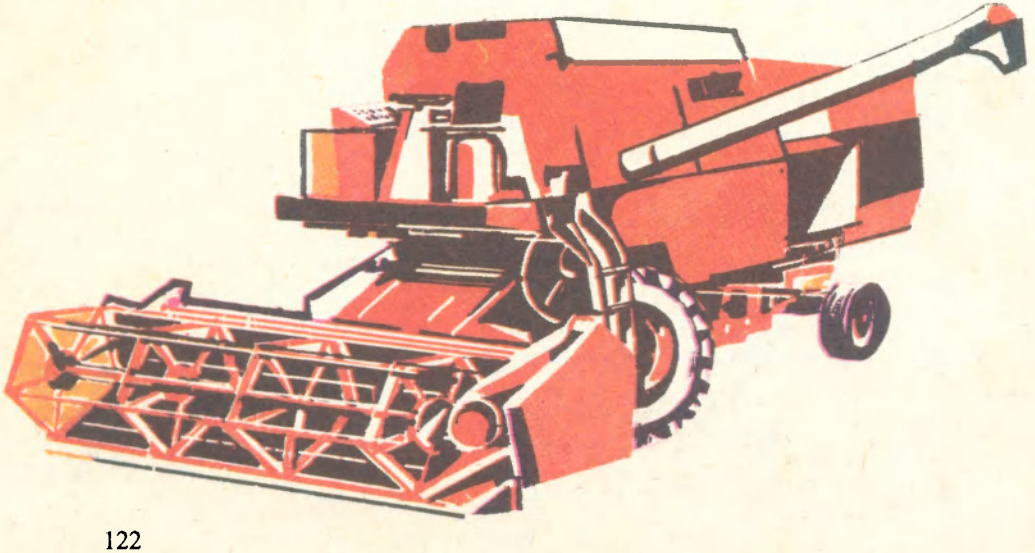
Where have you seen wheels that roll or spin?

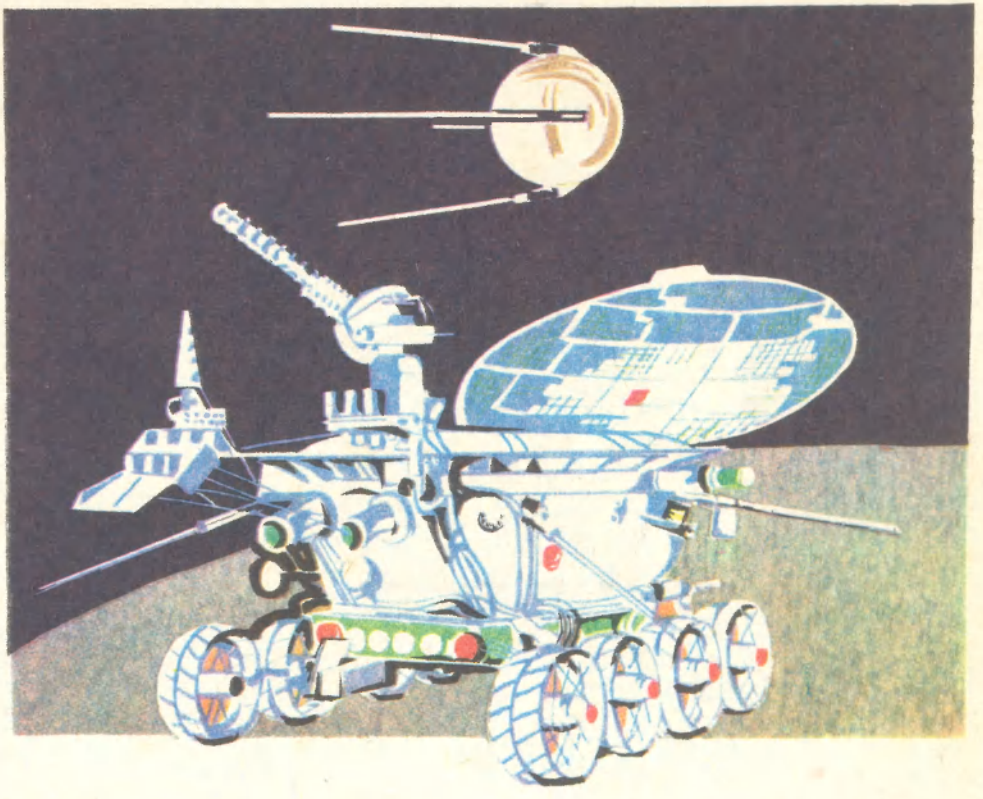


And just imagine a lathe at a factory or a turbine at a power plant.



There are so many various wheels there! Even in an ordinary clock, there're many wheels, small wheels and yet smaller wheels inside."





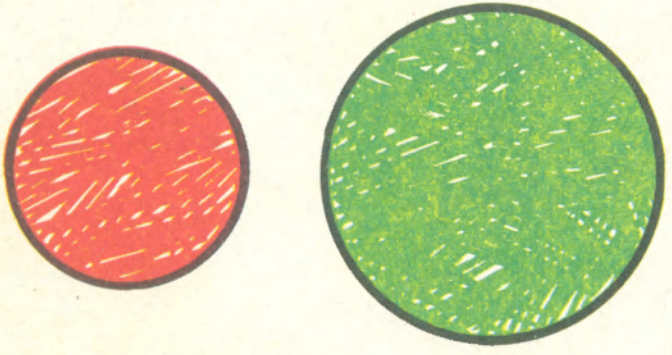
"You see thus often circles are used in engineering." Uncle John went on to say. Anybody who deals with machinery – workers, engineers, designers – must know all about geometry. For example, every design engineer knows that the axle of a wheel must pass through the centre of the circle. If a designer doesn't keep to this rule, he'll invent a car that won't roll."

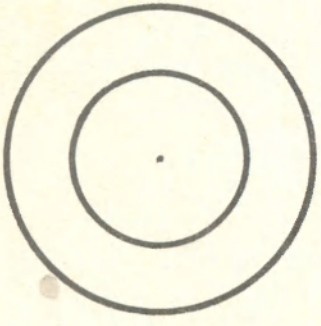
The gang listened to the engineer's story attentively. Gadgit was pleased most of all because he liked engineering very much. As he was listening to the designer, Gadgit made up his mind to take geometry seriously.

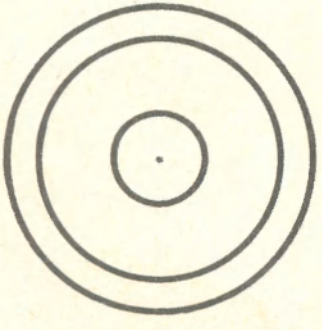
WORKSHEET

1

Here are two circles.
Which one is the larger, the red one or the green one?
Which one has the larger radius?

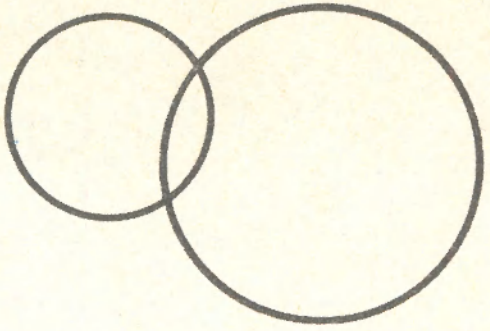






2

These two circles have the same centre.
And here are three circles with a common centre.
Now you draw some circles with the same centre.
You will have noticed that if you throw a stone into a pond with a smooth surface of water, some waves will propagate out in the form of circles with the same centre.

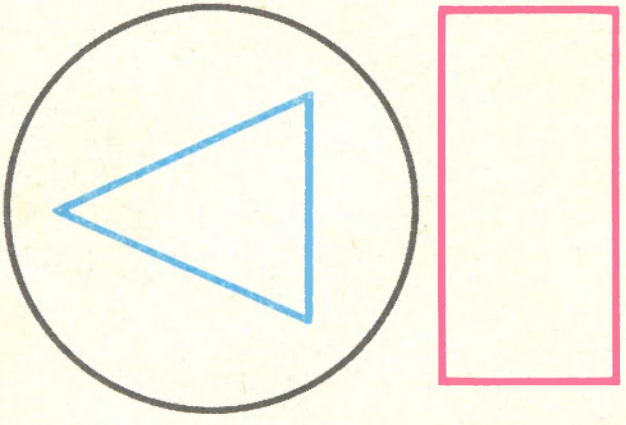


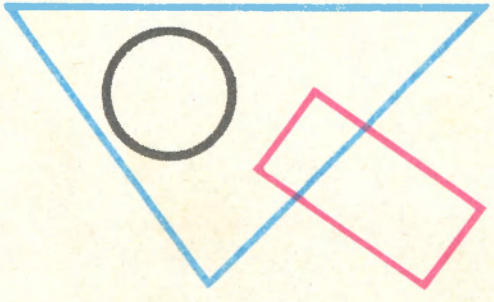
3

These circles intersect.
Point to the intersection points.
How many of them are there?
Now you draw two intersecting circles.

4

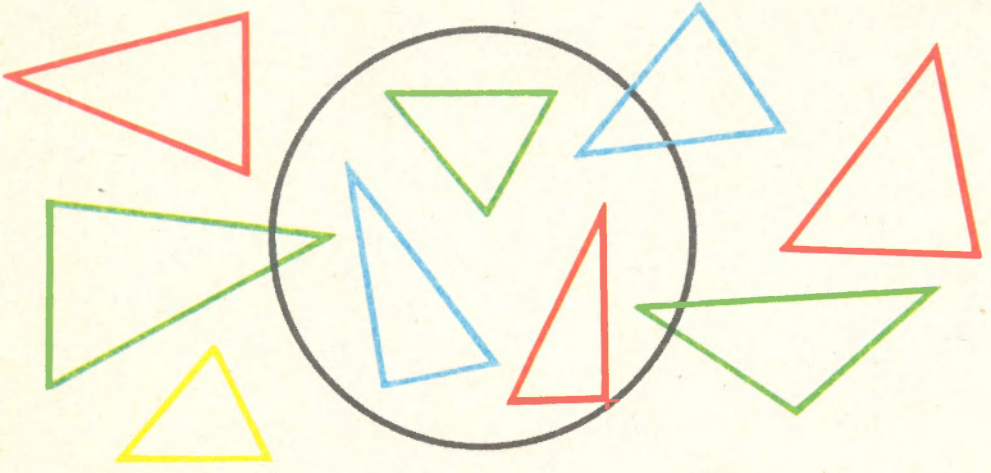
In this drawing the triangle lies inside the circle and the rectangle is completely outside the circle.

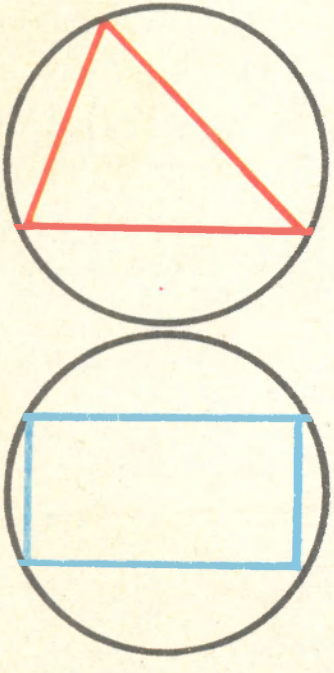




And here the circle is inside the triangle, but the rectangle intersects the triangle.

Count the triangles in this drawing that lie within the circle, the triangles that intersect the circle and the triangles that lie completely outside the circle.

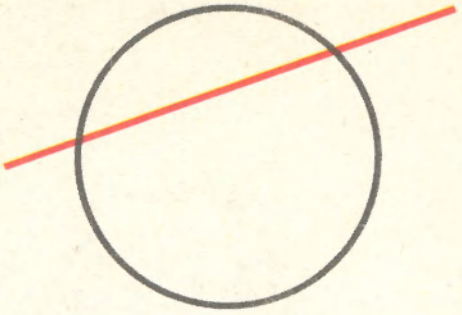




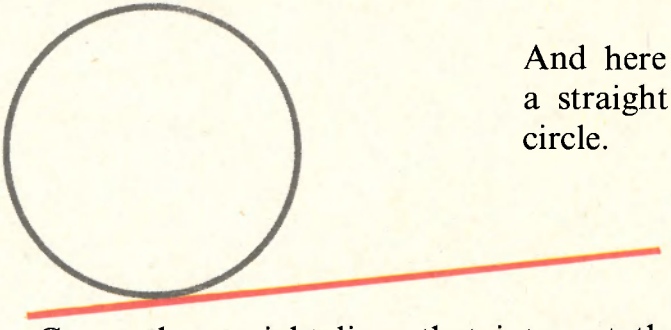
5

In this drawing the vertices of the triangle lie on the circumference. Such a triangle is said to be inscribed into the circle. Draw some circles with a triangle inscribed into each of them. And here is a rectangle inscribed in a circle. Now you try and draw a rectangle inscribed in a circle.

6

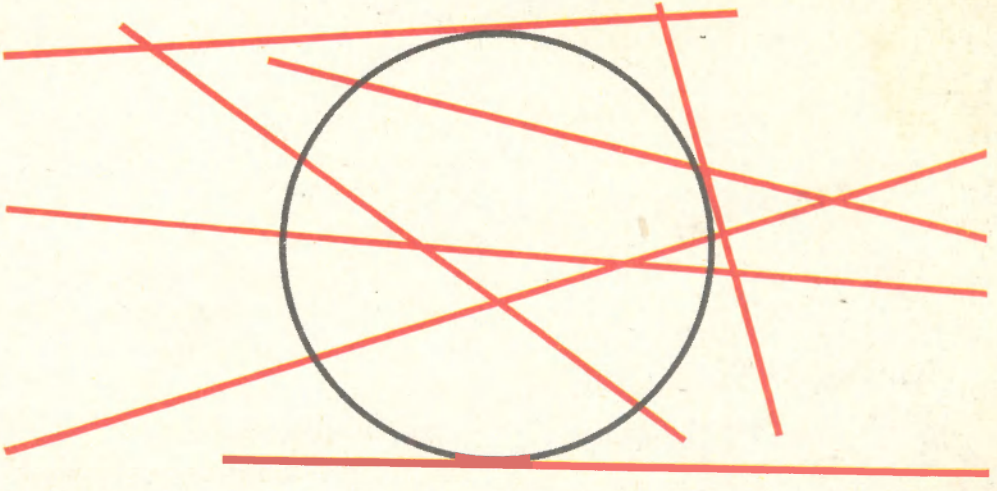


In this drawing a straight line intersects a circle.



And here
a straight line is tangent to a circle.

Count the straight lines that intersect the circle in this drawing, and those that are tangent to it.



Draw a circle, and then draw some straight lines that intersect it and some lines that are tangent to it.

When the Happy gang got together again, Pencil said:
"Well, my friends, now listen to the end of the fairy tale.

POINT'S
Travels in Geometry Land

...The plane started and again set out to search for Rubber-Robber.

Now all the passengers were looking for the wicked robber. Lot's of lines that were roads, rivers, and streams swept under the plane, and from time to time a town appeared in the distance. One town was directly under the plane.

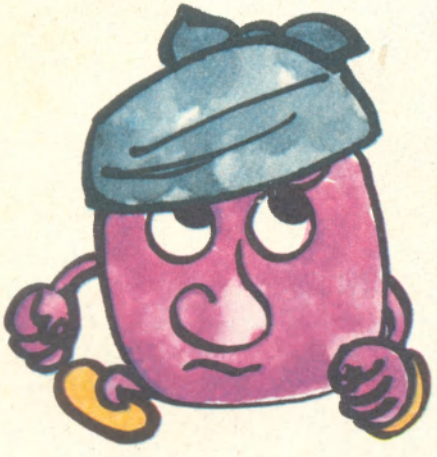
"Look, look!" cried Point, "there is another interesting town. Everything in it is round."

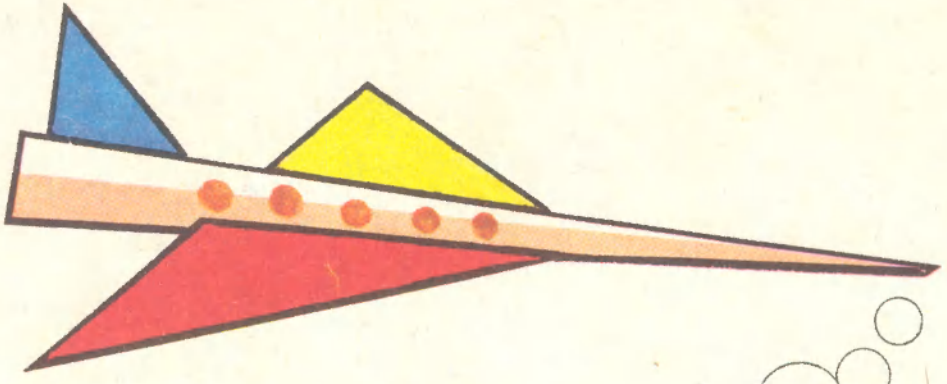
"Yes," said the triangles, "that is the Town of Circles. So everything must be round in it. Even books are round, and the books only have round letters."

Point wanted to find out more interesting things about the Town of Circles, but suddenly Compasses yelled:

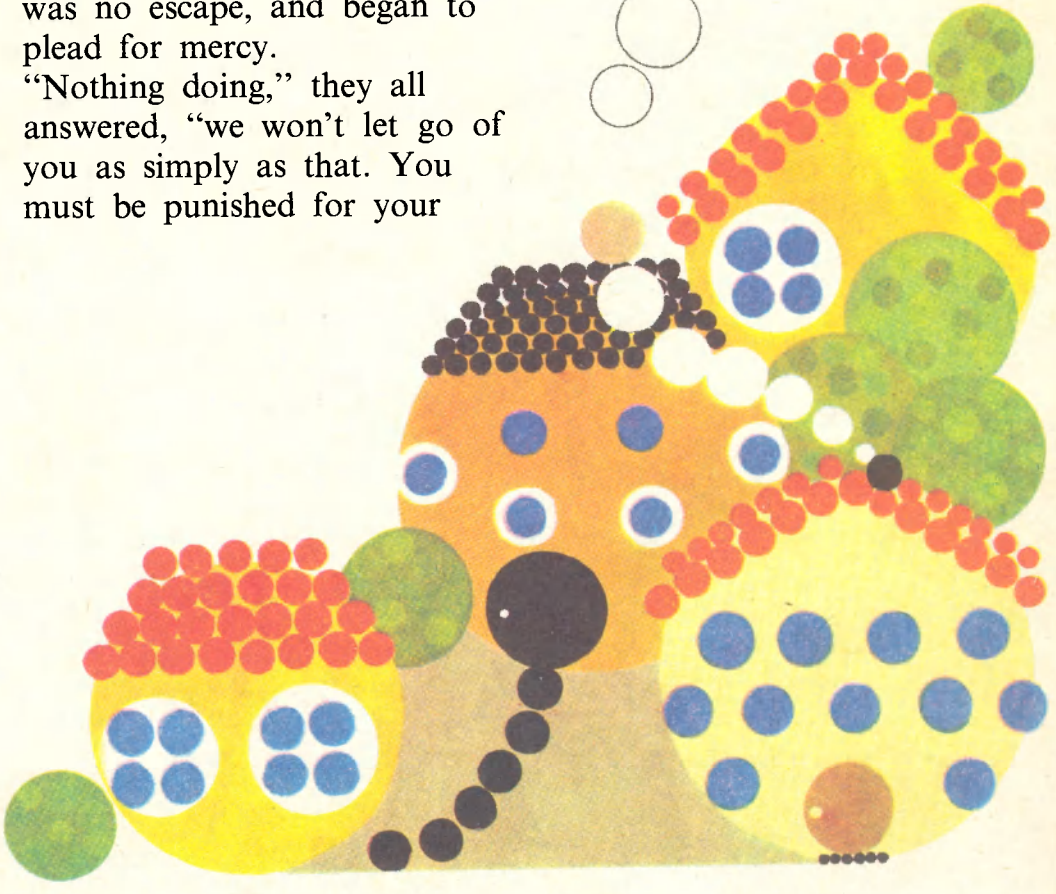
"I've just seen Rubber! He's over there, running along the road."

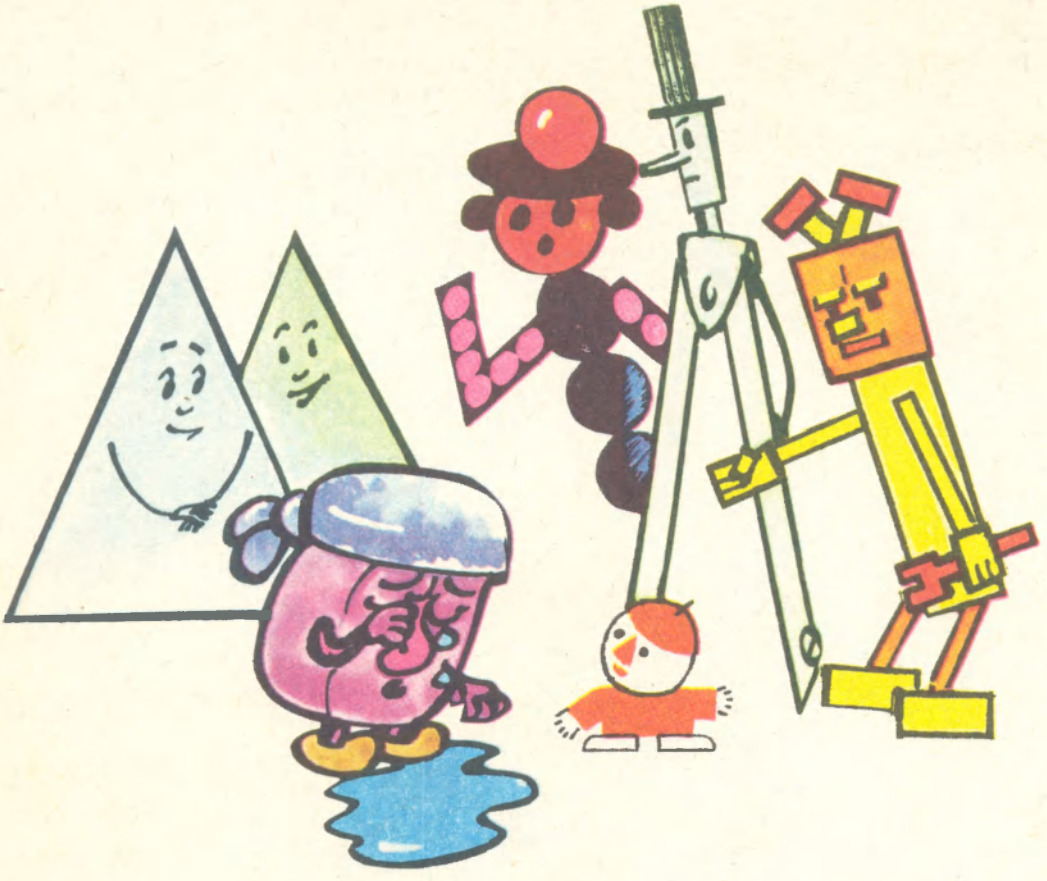
Rubber-Robber teared along, but the plane caught up with him and the pursuers jumped down using parachutes, landed and surrounded the villain.





Rubber-Robber saw that there was no escape, and began to plead for mercy.
"Nothing doing," they all answered, "we won't let go of you as simply as that. You must be punished for your

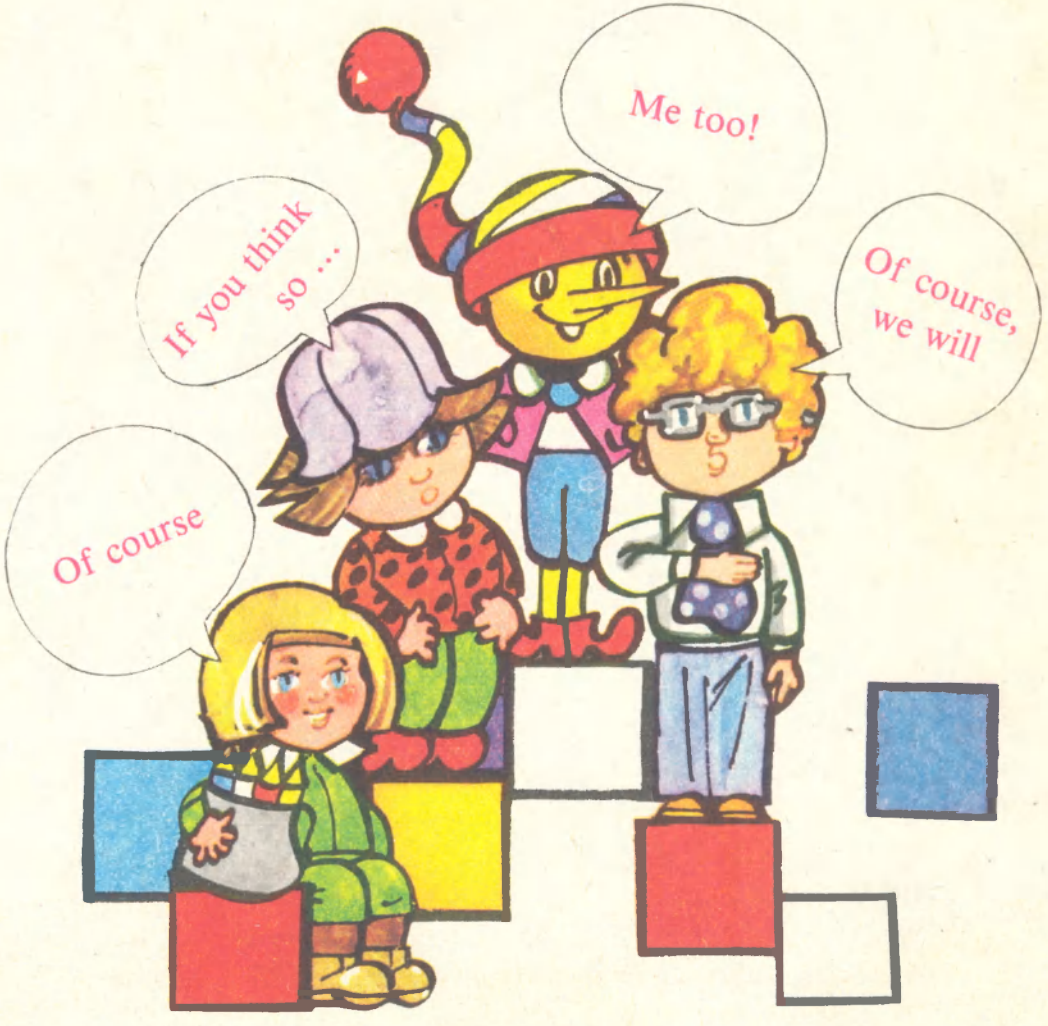




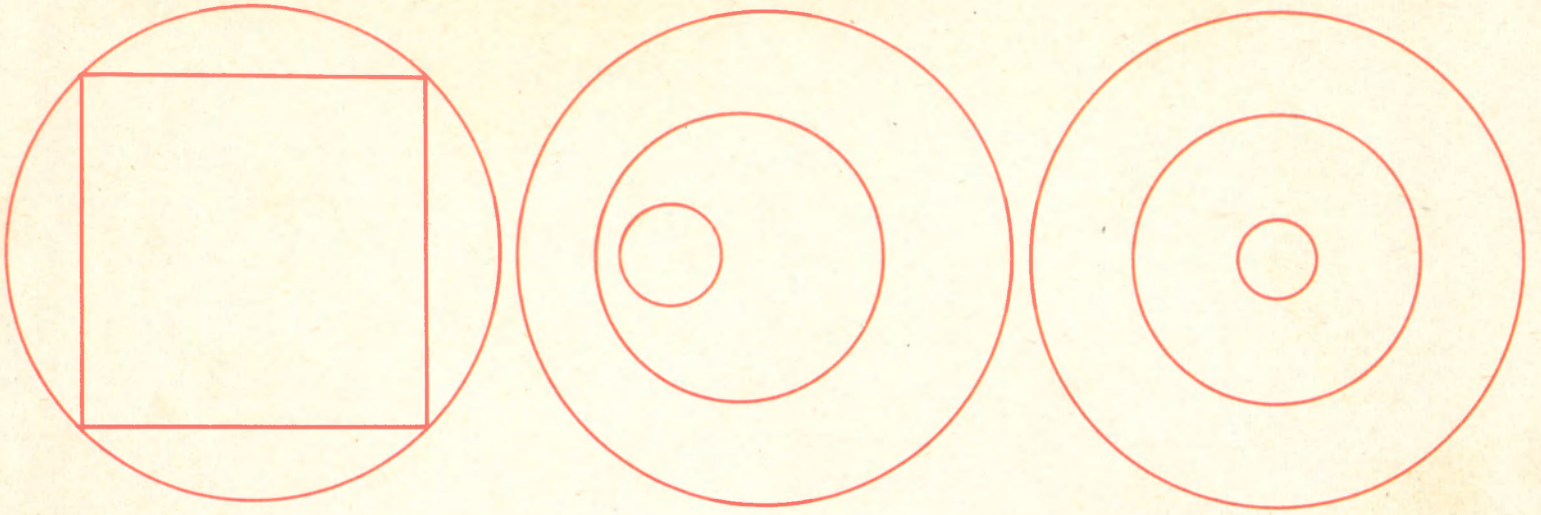
mischief. Now you see that straight line? Since you like erasing so much, you must erase it all."
And so Rubber-Robber set out to erase the straight line. He kept erasing and erasing and kept becoming smaller and smaller. He became quite small but could not yet erase the entire straight line.
"Spare me, let me go," he begged. "Never-never in my life will I do anything evil. I'll only erase when asked to."

"Okay then," said everybody, "we'll believe you. Go away." And so they let him go, and never since did he any more mischief in Geometry Land. He only erased something when somebody asked him.

"That's the end of the story," said Pencil, "and our classes have come to an end."

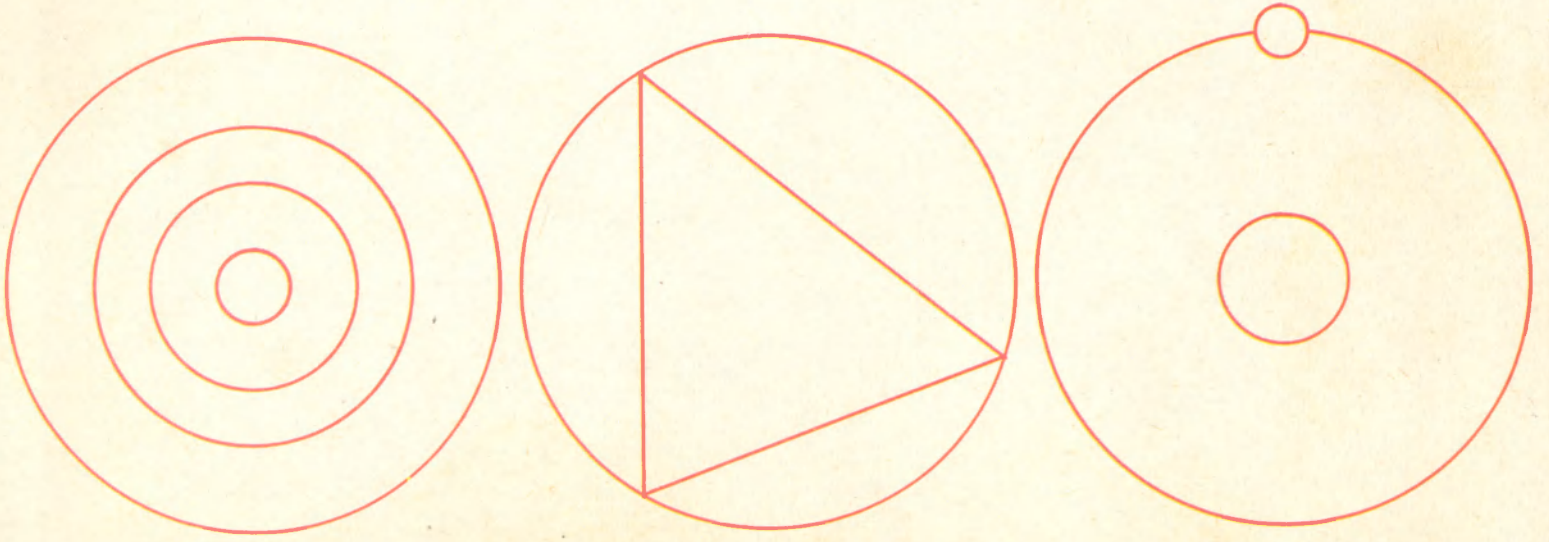


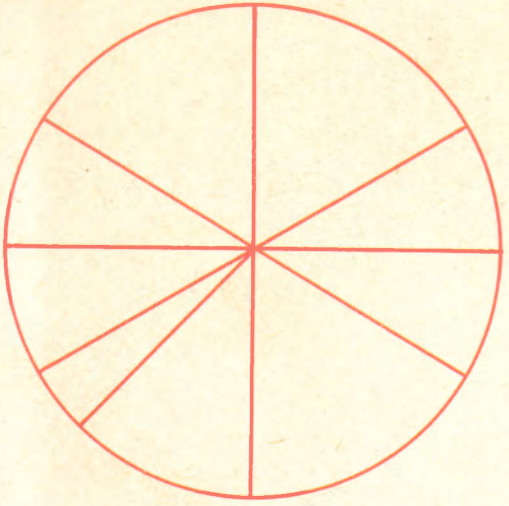
"Oh, have we learned everything about geometry?" Pinocchio was surprised.
"Why, Pinocchio! Of course not. Geometry is a very large science, and it takes a long time to master it."
"And shall we take it some time?"
"Of course, we will."

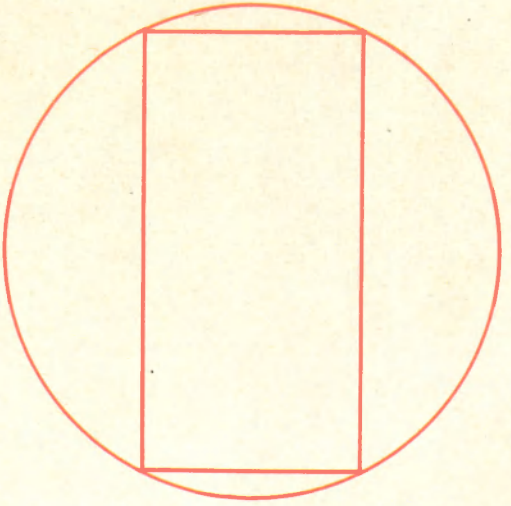


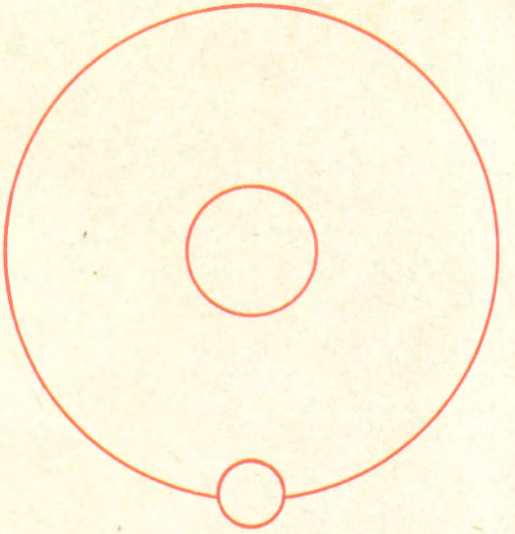
Shapes for colouring in

Shapes for colouring in









Shapes for colouring in

Through fascinating stories and rhymes, young readers are introduced to some elementary geometry.
The book is made up of adventures which bring in the theory, and some exercises to develop the topics.
It is written in a simple and attractive language, and is particularly well-suited to 5-8-year olds.